Supplier Selection with Multi-Scenario Inputs and Outputs, Using Data Envelopment Analysis

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Abstract

Data Envelopment Analysis (DEA) is a powerful tool for assessing the relative efficiency of Decision-Making Units (DMUs) by comparing their input-output ratios. Traditional DEA models typically address crisp data but may not fully capture scenarios involving uncertainty in inputs and outputs. This research expands on DEA by applying it to multiple scenarios to account for varying conditions in farms across Canada. This research applies DEA to assess the operational efficiency of greenhouse, sod, and nursery operations across Canada, using a multi-scenario approach over the period from 2019 to 2023. The study focuses on three key provinces, British Columbia, Ontario, and Quebec, making the sample size as 3, examining their performance based on various input variables, such as operating expenses for specialized greenhouse vegetables and flowers, and output variables, including the total value of greenhouse products produced. This model, derived from traditional DEA formulations but adapted for multiple scenarios, remains linear and computationally efficient, overcoming common limitations of non-linear models. Results indicate that Quebec consistently achieved full efficiency, while Ontario and British Columbia improved over time but did not reach Quebec's level. This study introduces a new application of DEA with the incorporation of a multi-scenario approach in order to capture the variation and uncertainty within the greenhouse, sod, and nursery operations in Canada which will improve the quality and strength of efficiency analyses relevant to agricultural settings. Despite its advancements, the study acknowledges limitations, including static analysis, homogeneity assumptions, and the exclusion of qualitative factors.

Keywords

Data Envelopment Analysis (DEA), Efficiency, Ranking, Multiple Scenario, Supplier Selection, Green House, Decision Making Unit (DMU)

Introduction

It is observed that the efficient selection of suppliers is a critical factor in gaining competitiveness and sustainability in the dynamic environment of supply chain management. Supplier selection is a process for assessing potential suppliers and choosing the most appropriate ones based on multiple criteria and constraints. Traditional methods are subjective, intuitive, or limited to some simple metrics, hence generally leading to poor decisions. Data Envelopment Analysis is one of the most commonly applied techniques in Operations and Supply Chain Management for measuring the relative efficiency of DMUs, like suppliers, based on several inputs and outputs. It is particularly useful when traditional methods will likely miss complexities and nuances in supplier selection within dynamic supply chain environments.

It does this by comparing the input-output ratios of each DMU to that of its peers to evaluate their relative efficiency. This technique simultaneously supports several inputs like cost, quality, and delivery time, as well as outputs such as customer satisfaction and product innovation. In doing this, DEA can give an overall assessment of supplier performance.

One of the critical advantages of DEA is its ability to deal with different scenarios and variations in inputs and outputs. This is critical in supply chain management, where factors like fluctuations in market demand, resource availability, or even regulatory changes may considerably impact supplier performance. In different scenarios, DEA helps establish strong choices of suppliers resistant to uncertainties and business environment fluctuations.

This research aims to investigate the application of DEA in supplier selection for the supply chain industry, considering multiple scenario inputs and outputs. By developing and implementing DEA models tailored to the complexities of supply chain dynamics, the study seeks to enhance the decision-making process, enabling organizations to make informed and robust supplier selections.

Supply Chain Management

The concept of a supply chain has changed many times and evolved through changes in business concepts, information technology, and globalization. The Council of Supply Chain Management Professionals (CSCMP) defines it as managing all procurement, production, and logistical services. It is about seamlessly knitting together the core business functions with other operational aspects across various stakeholder companies.

This collaboration involves suppliers, intermediaries, and customers synchronizing efforts across sales, marketing, finance, production, procurement, and logistics. Additionally, prior research, such as Tang (2006), characterizes SCM as coordinating material, information, and financial flows within a network of entities aiming to create and deliver consumer products or services.

The supply chain is a system of resources, processes, and stakeholders that manage the flow of materials, information, and finances from raw material sourcing to final product delivery to meet customer demand. It brings together different perspectives hitherto isolated by scholars: Athaudage et al. (2022) and Tang (2006).

Business activities are becoming more complex as globalization progresses. The value and reach of supply chains are increasing fast, and so are the associated stakeholders reaching out and engaging from all over the world. The supply chain is increasingly prone to disruption, and the market competition is getting tighter. All companies are eyeing strategies that will make them better placed in terms of efficiency as competitive forces rise. This is particularly true for multinational companies that are always looking to adapt and improve to remain competitive in the global market that continues to change. With the increasing significance of supply chains in enterprise operations, firms focus more on continuous improvement initiatives.

Supply chains are getting complex, and modern supply networks involve many participants across many industries and parts of the world. Various challenges arise when dealing with this highly competitive global market today (Vishnu et al., 2019). Many types of risks and threats, such as demand side, internal uncertainties, and supply, led to several pragmatic but prevalent and contemporary supply chain practices.

Companies have been suffering from several catastrophic events in the recent past caused by terrorist attacks, pandemics, and natural disasters, which broke or slowed down supply chains. Changing business trends, globalization, complexity, and specialization increase the risks but reduce managerial control of the operations (Urciuoli & Hintsa, 2018).

The concept of supply chains has its earliest discussions in logistics literature, dating back to the middle of the 20th century. The first contributions that brought out the integration of many different organizational functions to work out an efficient and low-cost operation in business were provided by Joseph Juran and W. Edwards Deming. However, their nature and focus were purely internal and did not extend to the network of suppliers, manufacturers, distributors, and customers.

During the 1980s, the decade we witnessed an astonishing movement toward the integrated approach to supply chain management. Concepts developed for Just-In-Time by Japanese manufacturers, especially Toyota, drew worldwide attention to the need to synchronize with customer demand, avoiding the high cost entailed by inventory carrying and storing. It also set the fundamentals of partnering between companies and their suppliers and would be the basis of modern-day supply chain partnerships.

We have seen extremely rapid growth in world trade in the second half of the 20th century. Transportation and communications improvements and trade liberalization have brought new challenges and complexities to supply chain management regarding extended lead times, stronger rivalry, and more significant supply chain risks. At this time, scholars and practitioners began to feel the need for more sophisticated strategies to deal with such challenges.

Since its inception, information technology has also contributed to changing the face of supply chain management. When ERP systems began to proliferate in companies in the 1990s, they provided better integration and smoother internal processes, making it easier to coordinate and communicate up and down the supply chain, as Davenport observed in 1998. It was also an era when concepts like Vendor-Managed Inventory and Collaborative Planning, Forecasting, and Replenishment took shape further to increase the visibility and responsiveness of supply chains.

The trends that appear to be building over the years are sustainability, digitalization, and omnichannel retailing. Sustainability and corporate social responsibility are fast being perceived as pertinent issues in the conduct of supply chain operations due to raised awareness by consumers and regulations (Seuring, 2012). Moreover, e-commerce and digital technologies have reset consumers' expectations toward ever-faster visibility, raising levels of personalization and seamless integrations between online and offline channels. (Chopra, 2015).

Data Envelopment Analysis

DEA is a method that calculates efficiency based on operational processes with multiple inputs and outputs solved within linear programming problems. Charnes et al. introduced this approach in 1978, transforming the multiple inputs and outputs into a single efficiency score. The process begins by identifying the best-performing units, forming what is known as the efficient frontier. Other units are then evaluated based on their proximity to this frontier.

Over time, DEA has evolved into various models like the CRS(constant returns to scale) and VRS(variable-returns-to-scale) models, each suited to different measurement needs. Its applications span industries such as banking, education, and healthcare (Avkiran, 2011). Furthermore, research on DEA extends globally, including countries like China (Zhong et al., 2011), Greece (Demirbag et al., 2010), and the UAE (Seiford & Thrall, 1990).

Various authors have examined the broad DEA literature and offered insights into the evolution of DEA methodology over different periods and from various perspectives. These surveys generally fall into bibliography listings, qualitative analyses, and quantitative assessments.

For instance, Seiford (1990) and Gattoufi et al. (2004) compiled extensive bibliographies of DEA literature. On the qualitative side, Seiford and Thrall (1990), Cooper et al. (2007), and Cook and Seiford (2009) offer in-depth discussions. For example, Seiford and Thrall (1990) delve into the early stages of DEA development, while Seiford (1996) tracks its evolution from 1978 to 1995, highlighting key milestones and conceptual shifts.

One notable aspect of Seiford's (1996) work is the visual evolution map, which graphically represents major events' timing and new ideas' emergence. This visualization aids in understanding the interconnectedness of ideas and tracing the genesis of novel concepts.

From a theoretical standpoint, Cooper et al. (2007) analyze various DEA models and metrics, while Cook and Seiford (2009) offer a comprehensive review of methodological advancements since 1978. Their review covers various topics, including generic DEA models, multilevel models, constraints on multipliers, variable status considerations, and handling of data variations.

Gattoufi et al. (2004) and Emrouznejad et al. (2008) conducted quantitative surveys and provided analyses of publication statistics using DEA. Gattoufi et al. (2004) sourced their data from six online professional databases, while Emrouznejad et al.(2008) maintained a regularly updated DEA literature database with community support.

As already pointed out, the DEA model was introduced as far back as 1978 by Charnes, Cooper, and Rhodes, though Farrell had put forth the efficiency concept as early as 1957. First, the CCR model, later extended in 1984 by Banker, Charnes, and Cooper to yield the so-called Banker, Charnes, and Cooper model, introduced constant returns to scale and variable returns to scale concepts, respectively.

Today, DEA has become an essential tool for decision-making, helping to evaluate efficiency and identify production capabilities. DEA is a methodology based on linear programming used to determine the production efficiency of suppliers by employing multiple input and output variables (Hosseini-Nasab & Ettehadi, 2023). This approach allows for distinguishing vendors based on their efficiency levels. DEA is a mathematical method for calculating an economic unit's relative productivity or efficiency, enabling the measurement of efficiency for a set of Decision-Making Units (DMUs). A DMU refers to a homogeneous entity or productive unit offering similar products or services.

Additionally, DMUs can produce multiple output variables based on various input variables. DEA's mathematical programming technique helps identify which DMU has the highest efficiency score, aiding supplier selection.

After defining a set of DMUs, the next step is to specify the input and output variables. The DEA model accommodates multiple input and output variables (Cikovic et al., 2022), and the number of input variables can differ from the number of output variables. In DEA, all DMUs use the same input and output variables.

To assess each DMU's efficiency, a weighted ratio is assigned to each input and output variable for each DMU. By applying these weight ratios, the efficiency rate of each DMU can be calculated under optimal conditions to achieve maximum efficiency. The efficiency rate of a DMU can be expressed as the weighted sum of outputs divided by the weighted sum of inputs.

Maximize

$$\theta_q = \sum_{k=1}^{r} u_k y_{kq}$$

Subject to;

$$\sum_{k=1}^{r} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, 2, ..., n,$$

$$\sum_{i=1}^{m} v_i x_{iq} = 1,$$

$$u_k \geq \varepsilon, v_i \geq \varepsilon.$$

v_i – Input weight of the ith input

u_k – Output weight of the kth output

Non-Archimedean element $\varepsilon > 0$,

v_i and u_k represent input and output weights for the i-th input and k-th output. On the other hand, $\varepsilon > 0$ is a non-Archimedean element, smaller than any positive real number.

The optimal objective function value q_q*=1 demonstrates the efficiency of the evaluated unit. Units with values less than one are considered inefficient. Reducing the number of inputs can help achieve the efficiency threshold (Cengage, 2022).

A set of input and output variables is necessary to determine the efficiency score using DEA. Inputs are any resources consumed by a DMU, while outputs are the results or performance measures of transforming these inputs into products or services (Wong, 2021).

The choice between input and output orientation depends on the objective. An input-oriented DEA model examines the capability to produce a given output level with the least amount of inputs and resources (Alidrisi, 2021). Conversely, an output-oriented DEA model evaluates how effectively a DMU maximizes its output with a fixed level of inputs.

Consider Ranch House, Inc., a fast-food restaurant chain with operations in five different locations. To determine the most efficient restaurant in the chain, they have collected input and output data to develop a Data Envelopment Analysis (DEA) model. The inputs measured include weekly hours of operation, the number of full-time equivalent staff, and weekly supply expenses. The outputs are the average weekly contribution to profit, market share, and annual growth rate. Tables 1 and 2 below display the collected input and output data, respectively (Cengage, 2022).

Table 1- Input measures

Restaurant	Hours of Operation	FTE Staff	Supplies ($)
Bardstown	96	16	850
Clarksville	110	22	1400
Jeffersonville	100	18	1200
New Albany	125	25	1500
St. Matthews	120	24	1600

Table 2 - Output measures

Restaurant	Weekly Profit ($)	Market Share(%)	Growth Rate (%)
Bardstown	3800	25	8.0
Clarksville	4600	32	8.5
Jeffersonville	4400	35	8.0
New Albany	6500	30	10.0
St. Matthews	6000	28	9.0

Developing the Mathematical Model for each DMU:

DEA model for Bardstown:

$Max\ \theta = 3800u_1 + 25u_2 + 8u_3$

Subject to

$$3800u_1 + 25u_2 + 8u_3 - 96v_1 - 16v_2 - 850v_3 \leq 0$$
$$4600u_1 + 32u_2 + 8.5u_3 - 110v_1 - 22v_2 - 1400v_3 \leq 0$$
$$4400u_1 + 35u_2 + 8u_3 - 100v_1 - 18v_2 - 1200v_3 \leq 0$$
$$6500u_1 + 30u_2 + 10u_3 - 125v_1 - 25v_2 - 1500v_3 \leq 0$$
$$6000u_1 + 25u_2 + 9u_3 - 120v_1 - 24v_2 - 1600v_3 \leq 0$$
$$96v_1 + 16v_2 + 850v_3 = 1$$
$$u_1, u_2, u_3, v_1, v_2, v_3 \geq \varepsilon$$

DEA model for Clarksville:

$Max\ \theta = 4600u_1 + 32u_2 + 8.5u_3$

Subject to

$$3800u_1 + 25u_2 + 8u_3 - 96v_1 - 16v_2 - 850v_3 \leq 0$$
$$4600u_1 + 32u_2 + 8.5u_3 - 110v_1 - 22v_2 - 1400v_3 \leq 0$$
$$4400u_1 + 35u_2 + 8u_3 - 100v_1 - 18v_2 - 1200v_3 \leq 0$$
$$6500u_1 + 30u_2 + 10u_3 - 125v_1 - 25v_2 - 1500v_3 \leq 0$$
$$6000u_1 + 25u_2 + 9u_3 - 120v_1 - 24v_2 - 1600v_3 \leq 0$$
$$110v_1 + 22v_2 + 1400v_3 = 1$$
$$u_1, u_2, u_3, v_1, v_2, v_3 \geq \varepsilon$$

DEA model for Jeffersonville:

$Max\ \theta = 4400u_1 + 35u_2 + 8u_3$

Subject to

$$3800u_1 + 25u_2 + 8u_3 - 96v_1 - 16v_2 - 850v_3 \leq 0$$
$$4600u_1 + 32u_2 + 8.5u_3 - 110v_1 - 22v_2 - 1400v_3 \leq 0$$
$$4400u_1 + 35u_2 + 8u_3 - 100v_1 - 18v_2 - 1200v_3 \leq 0$$
$$6500u_1 + 30u_2 + 10u_3 - 125v_1 - 25v_2 - 1500v_3 \leq 0$$
$$6000u_1 + 25u_2 + 9u_3 - 120v_1 - 24v_2 - 1600v_3 \leq 0$$
$$100v_1 + 18v_2 + 1200v_3 = 1$$

$$u_1, u_2, u_3, v_1, v_2, v_3 \geq \varepsilon$$

DEA model for New Alberny:

$Max\ \theta = 6500u_1 + 30u_2 + 10u_3$

Subject to

$$3800u_1 + 25u_2 + 8u_3 - 96v_1 - 16v_2 - 850v_3 \leq 0$$
$$4600u_1 + 32u_2 + 8.5u_3 - 110v_1 - 22v_2 - 1400v_3 \leq 0$$
$$4400u_1 + 35u_2 + 8u_3 - 100v_1 - 18v_2 - 1200v_3 \leq 0$$
$$6500u_1 + 30u_2 + 10u_3 - 125v_1 - 25v_2 - 1500v_3 \leq 0$$
$$6000u_1 + 25u_2 + 9u_3 - 120v_1 - 24v_2 - 1600v_3 \leq 0$$
$$125v_1 + 25v_2 + 1500v_3 = 1$$
$$u_1, u_2, u_3, v_1, v_2, v_3 \geq \varepsilon$$

DEA model for St. Matthews:

$Max\ \theta = 6000u_1 + 25u_2 + 9u_3$

Subject to

$$3800u_1 + 25u_2 + 8u_3 - 96v_1 - 16v_2 - 850v_3 \leq 0$$
$$4600u_1 + 32u_2 + 8.5u_3 - 110v_1 - 22v_2 - 1400v_3 \leq 0$$
$$4400u_1 + 35u_2 + 8u_3 - 100v_1 - 18v_2 - 1200v_3 \leq 0$$
$$6500u_1 + 30u_2 + 10u_3 - 125v_1 - 25v_2 - 1500v_3 \leq 0$$
$$6000u_1 + 25u_2 + 9u_3 - 120v_1 - 24v_2 - 1600v_3 \leq 0$$
$$120v_1 + 24v_2 + 1600v_3 = 1$$
$$u_1, u_2, u_3, v_1, v_2, v_3 \geq \varepsilon$$

The following table shows the results of implementing and solving the above mathematical models using Excel. (Cengage, 2022).

Table 3 - Optimal Efficiency Score

Restaurant	Optimal Efficiency Score
Bardstown	1
Clarksville	0.96
Jeffersonville	1
New Albany	1
St. Matthews	0.99

An efficiency score of 1 represents optimal efficiency, while a score less than 1 indicates inefficiency. We can rank DMUs based on their efficiency scores from highest to lowest. (Zahedi-Seresht et al., 2023b)

There are no strict guidelines for determining input and output variables in a DEA model. Operational indicators such as total assets, capital, current liabilities, operating expenses, number of staff, and overhead expenses can be treated as input variables. Likewise, operating income, net profit, net sales, or revenue can be viewed as output variables (Wong, 2021) when using the DEA method to calculate the efficiency score for comparative purposes.

Despite its foundational role, traditional DEA models like CCR and BCC often rely on crisp data representations, overlooking the prevalent uncertainty in real-world datasets. O'Neal, Ozcan, and Yanqiang (2002) suggested excluding data with uncertainties, impacting the relative efficiency scores. Alternative approaches, like imputation techniques, have been proposed, yet they risk distorting efficiency scores.

Stochastic programming, dating back to Dantzig (1955) and Beale (1955), offers another avenue for uncertain data in DEA, albeit challenged by the lack of empirical evidence for probability distribution functions. Kuosmanen (2009) proposed using dummy variables for missing data, but this method has limitations in accurately reflecting reality.

Fuzzy methodologies, pioneered by Sengupta (1992) and Cooper, Park, and Yu (1999), offer a promising avenue by accommodating uncertain data through fuzzy logic and interval-based approaches. Despite challenges in assessing upper and lower bounds of efficiency, researchers like Despotis and Smirlis (2002) and Kao (2006) have made strides in developing interval methods.

Fuzzy DEA, explored by researchers like Guo and Tanaka (2001) and Hatami-Marbini and Saati (2009), offers an intricate yet effective means of evaluating DMU efficiency amidst uncertainty. Despite the method's complexity, it holds promise for practical applications.

In recent years, scholars like Toloo and Ertay (2014) have introduced novel approaches like CE-DEA to address uncertainties in input prices, while Sadjadi and Omrani (2008) proposed robust DEA models for evaluating performance under output uncertainty. Shokouhi et al. (2010) and Hafezalkotob et al. (2015) have also contributed robust optimization models to address uncertainty in input and output parameters.

Further advancements include methods proposed by Arabmaldar, Jablonsky, and Saljooghi (2017) and Zahedi-Seresht, Mehrabian, and Jahanshahloo (2016), aiming to enhance efficiency evaluation in uncertain environments.

While traditional DEA models paved the way, emerging methodologies like fuzzy DEA and robust optimization offer promising avenues for addressing uncertainty and enhancing efficiency evaluations in diverse decision-making contexts. However, challenges remain in balancing computational complexity with real-world applicability.

Data Envelopment Analysis in Supply Chain

Supply chain management refers to coordinating material, information, and financial flows that satisfy customers' demands efficiently and at low costs, maximizing the supply chains' overall profitability. Data envelopment analysis has been an extremely fundamental mathematical tool in assessing performance in the supply chain over the years. Despite numerous DEA models proposed for this purpose, systematic literature reviews are lacking.

Researchers have conducted a comprehensive review using the PRISMA method to address this gap, analyzing 75 articles from 1996 to 2016 across 35 scholarly journals and conferences. They categorized these articles based on author, publication year, technique, application area, country, scope, purpose, research gap, contribution, and outcome. (Soheilirad et al., 2017).

The findings highlight that supplier selection, supply chain efficiency, and sustainable practices are the most frequently studied areas. DEA proves promising as an evaluative tool, especially when traditional production function data is scarce or difficult to obtain. Its ability to handle multiple inputs and outputs makes it attractive for researchers in supply chain management and various organizational and industrial contexts.

For example, Li et al. (2012) have used DEA to measure the supply chain efficiency of the Chinese manufacturing industry. In their findings, DEA highlights where the chain is inefficient and provides suggestions on how it can be enhanced for leaner and more cost-effective supply chain operations.

Furthermore, in sustainable supply chain practices, Wu et al. (2013) used DEA to measure the environmental efficiency of supply chains. Their research illustrated how DEA could integrate environmental factors into the efficiency analysis, promoting sustainable practices by identifying which supply chains achieve a balance between economic and environmental performance.

Furthermore, Chen et al. (2016) applied DEA to measure the operational efficiency of logistics companies from a supply chain perspective. Their findings further underscored the ability of DEA to pinpoint those logistics companies with the opportunity and potential to improve their operations to improve the supply chains' overall efficiency.

A more advanced application is reported in an article published by the European Journal of Operational Research, where a two-stage DEA model was applied to evaluate the performance of a three-tier supply chain involving suppliers, manufacturers, and distributors. This approach helps measure efficiency with respect to the whole chain and various components at every level; therefore, it does a more detailed analysis. (Tavana, 2015).

Another significant study in the International Journal of Systems Science explored how DEA can be adapted to consider environmental factors in supply chain performance evaluation. Their model incorporates undesirable outputs, such as emissions or waste generation, alongside traditional metrics to provide a holistic assessment that integrates sustainability (Liu et al., 2012).

In conclusion, DEA is, in general, a potent tool in supply chain management, as it can provide a very robust framework for evaluating and improving performance across several dimensions, such as supplier selection, overall efficiency, and sustainability. Its flexibility and the possibility of handling multiple inputs and outputs make it an invaluable resource for researchers and practitioners seeking to optimize supply chain operations.

Supplier Selection Using Data Envelopment Analysis

Supplier evaluation is a heavily researched topic in the field of purchasing, with various methodologies ranging from theoretical to empirical and modeling approaches. This paper will not cover all these studies in detail, as it focuses on quantitative models for supplier evaluation.

Empirical research about supplier evaluation emerged in the 1960s. In this regard, a seminal study was published by Dickson (1966), which analyzed the factors that industrial purchasing managers perceive as relevant in supplier evaluation. The findings highlighted cost, quality, and delivery performance as the top three criteria. Subsequent research has supported this and developed it further, all about the strategic importance of supplier evaluation and the trade-offs between cost, quality, and delivery. Other studies have focused on the issue of the relative importance of supplier attributes. Monczka et al. (1981), Moriarity (1983), Woodside and Vyas (1987), Chapman and Carter (1990), Tullous and Munson (1991), and Weber et al. (1991) are some of the studies conducted toward this end.

In their review of 74 articles, Weber et al. (1991) found that quality was the most critical factor, followed by delivery performance and cost. These studies collectively indicate that supplier evaluation should consider multiple factors rather than focusing solely on cost. However, they have not developed specific decision models for supplier evaluation.

In a thorough review of supplier selection methods, Weber et al. (1991) found that 47 out of 74 articles reviewed used multiple criteria. Traditional multi-criteria approaches have included factors like cost, quality, and delivery, which have gained importance with the focus on Just-In-Time (JIT) manufacturing philosophy (Chapman, 1989; Chapman & Carter, 1990). However, these measures are mainly operational. Table 4 shows the supplier evaluation techniques categorized by methodological area. Table 4 shows the supplier selection techniques followed by different methodologies.

Table 4 -Supplier Selection and Evaluation Techniques

Evaluation technique	**Authors**
Grouping methods	Hinkle et al. (1969)
Weighted linear models	Lamberson et al. (1976)
Weighted linear models	Timmerman (1986)
Matrix method	Gregory (1986)
Analytic hierarchy process	Narasimhan (1983)
Linear programming	Turner (1988)
Linear programming	Pan (1989)

Analytic hierarchy process	Hill and Nydick (1992)
Mixed integer programming	Weber and Current (1993)
Multi-objective programming	Weber and Ellram (1993)
Interpretive structural modeling	Mandal and Deshmukh (1994)
Total cost of ownership	Ellram (1995)
DEA	Weber and Desai (1996)
Statistical analysis	Mummalaneni et al. (1996)
Analytic hierarchy process	Barbarosoglu and Yazgac (1997)
Neural networks	Siying et al. (1997)
DEA	Weber et al. (1998)
Discrete choice analysis experiments	Verma and Pullman (1998)
Principal component analysis	Petroni and Braglia (2000)
DEA	Narasimhan et al. (2001)

As depicted in Table 4, Several techniques have employed multiple criteria for evaluating suppliers. However, many of these techniques face issues such as the absence of objective methods for assigning factor weights, the lack of relative comparisons of alternative suppliers for benchmarking and Supplier Development Initiatives (SDI), insufficient emphasis on strategic level capabilities or practices, and not addressing the issues and reasons behind ineffective supplier performance.

The use of Data Envelopment Analysis (DEA) as a tool for strategic sourcing of suppliers has been limited. Few studies have applied DEA for supplier evaluation. For instance, Kleinsorge et al. (1992) used DEA for performance monitoring of a single supplier over time, but their research did not address strategic supplier selection or benchmarking issues. Weber and Desai (1996) and Weber et al. (1998) addressed supplier selection and negotiation using DEA, but they focused solely on operational metrics and relied on a traditional DEA model, which has certain limitations. Narasimhan et al. (2001) applied DEA for the strategic evaluation of suppliers by considering various strategic and operational factors. While their approach offered useful insights into supplier evaluation and rationalization, it was also constrained by traditional DEA model evaluations. Additionally, their work did not explore the reasons behind differences in suppliers' efficiency scores and therefore did not propose supplier improvement strategies

A study by Talluri and Narasimhan (2004), DEA was employed to assess the performance of suppliers by considering multiple input and output criteria. The study revealed that DEA could effectively identify efficient suppliers and provide insights into improving the performance of inefficient ones, demonstrating its practical applicability in supplier selection processes. The study was conducted for a major global telecommunications company. Several meetings were held to determine the specific product line to be analyzed and to identify the input and output dimensions for Data Envelopment Analysis (DEA).

Inputs include Quality management practices and systems (QMP), Documentation and self-audit (SA), Process/manufacturing capability (PMC), Management of the firm (MGT), Design and development capabilities (DD), Cost reduction capability (CR). The outputs included Quality, Price, Delivery, Cost reduction performance (CRP) and Other factors. A sample of the data structure is depicted in Table 5 as follows.

Table 5- Data Structure of the DEA used by Talluri and Narasimhan (2004) for supplier selection research for a large telecommunication entity.

Scaled supplier data with inputs and outputs and efficiency scores

Supplier #	QMP	SA	PMC	MGT	DD	CR	Quality	Price	Delivery	CRP	Other	CCR Eff.	X-Eff. mean	Standard deviation
1	0.9662	0.9742	1.0385	1.0808	1.1417	0.7839	0.6211	0.8922	0.1284	1.2107	0.6359	0.602	0.427	0.129
2	0.7054	1.0438	0.7500	0.8782	0.0000	0.8750	0.6932	0.8922	0.3855	0.0000	0.3179	1.000	0.412	0.288
3	0.5611	0.8947	0.7789	0.7205	0.8372	0.7404	1.0205	0.4341	1.5420	0.0000	1.2719	1.000	0.536	0.326
4	1.1272	1.0438	0.9520	0.9607	0.9661	1.1402	1.6639	1.1333	1.5420	1.2107	1.8019	1.000	0.752	0.243
5	1.1272	1.0438	1.1251	1.0808	1.2560	1.2115	0.9983	1.3503	1.1565	1.2107	0.9540	0.855	0.615	0.207
6	0.9877	1.0438	0.9376	1.0808	1.0466	0.9422	1.0426	1.3263	1.7990	2.4214	1.2719	1.000	0.810	0.171
7	0.8051	0.8351	1.0385	0.9607	1.2560	1.0768	1.2201	1.2056	0.7710	2.4214	1.2719	1.000	0.821	0.207
8	1.1809	1.0438	1.1251	1.0208	1.0627	1.0096	0.8429	1.1333	0.6424	1.2107	0.8479	0.723	0.523	0.156
9	1.2346	1.0438	1.1251	1.0808	1.2560	1.1442	0.6433	0.8922	0.3855	0.0000	0.5299	0.562	0.316	0.201
10	0.5904	1.0438	0.6058	0.7629	0.5796	0.4038	1.4419	0.4341	1.4135	0.0000	1.2719	1.000	0.578	0.369

Source: Talluri, S., & Narasimhan, R. (2004). A methodology for strategic sourcing. European Journal of Operational Research, 154(1), 236–250

One limitation of the study is that it considered only one respondent for each supplier. This approach may not capture the full spectrum of perspectives and experiences related to the supplier's performance. Therefore, this study may have a bias of only one respondent or incomplete information since this is also the view and knowledge of one respondent and not a comprehensive assessment by different stakeholders of the supplier organization. Thus, this may influence the accuracy and reliability of the findings and conclusions of the present study.

A study by Zahedi-Seresht et al. (2017) proposed using the area's magnitude under the efficient curve. In order to estimate this magnitude, they suggest using Monte Carlo simulation for the complete ranking of originally efficient DMUs to overcome the problems arising from other ranking methods, which is very simple, computationally. This method generates random weights for the inputs and outputs in the feasible region and finally derives probability the DMUs are efficient.

Another research by Jahanshahloo & Zahedi Seresht, (2015), utilizes monte carlo method for ranking extreme efficient units in data envelopment analysis. They highlighted that there have been so many attempts in the literature which have their pros and cons in ranking extreme efficient units. They attempt to present a method which does not have any problems and can be utilized to calculate the rank of extreme efficient units through using the Hit or Miss Monte Carlo method

Multi-Scenario Data Envelopment Analysis in Supply Selection

The previous section shows that supplier selection in supply chain management is very important. If properly done, it can influence an organization's ability to be competitive and sustainable. Traditionally, supplier selection has been approached using different methods, most of which are normally based on some simplistic metrics or subjective evaluation. Modern-day supply chains are usually complex, while the traditional assessment methods are too inadequate to capture the multidimensionality of supplier performance. A major problem in supplier selection could be that multiple scenarios exist for both inputs and outputs within DMUs.

Multiple responses for inputs and outputs in DMUs are commonplace due to several factors inherent in supply chain dynamics. Firstly, suppliers may offer various products or services with characteristics and attributes. For instance, a supplier may deliver multiple raw materials or components that would differ by quality, cost, and lead time. Alternatively, suppliers can also have a good many customer segments they sell to, or even operate in various geographic regions, so their performance may vary across different markets or product lines at different points in time.

Multiple responses pose challenges and opportunities in supplier selection using Data Envelopment Analysis (DEA). DEA is a non-parametric method that evaluates the relative efficiency of DMUs based on their input-output relationships. When multiple responses are available for inputs and outputs, DEA allows for the simultaneous consideration of these diverse dimensions, providing a comprehensive assessment of supplier performance.

For example, a manufacturing company is to choose the suppliers for one of its critical components. That company will have to evaluate potential suppliers against multiple criteria, including cost, delivery time, quality, and flexibility. Each supplier may have different performance levels across different product lines or regions for each criterion. For example, a Supplier might offer lower costs but longer lead times in one region while providing high-quality goods. Conversely, the same supplier might offer a high cost and short lead time with the same quality for the goods. So there will be two costs values and two lead times which we have to taken in to account when we incorporate the responses for DEA.

In supply chain management, DMUs such as suppliers may exhibit variability or multiple responses in their performance for a single input or output due to various factors. For example, a supplier may offer different price points for the same product or service, reflecting variations in negotiation strategies, market conditions, or production costs. Similarly, a supplier's delivery performance may vary across orders or customers, influenced by transportation logistics, inventory management, or order complexity.

For example, supplier price variability is a common phenomenon influenced by negotiation tactics, market dynamics, and production costs. Chen and Paulraj (2004) highlight that suppliers, as a single decision-making unit, may adopt different pricing strategies based on customer segments, order quantities, and competitive pressures. This pricing variability reflects not merely cost structures but also strategic decisions made to maximize competitive advantage and market share. For example, a supplier might offer lower prices to large-volume buyers or strategically essential customers, while higher prices may be quoted to smaller, less frequent buyers. This variability impacts a firm's procurement decisions and overall supply chain costs, necessitating sophisticated price analysis, and negotiation strategies to optimize procurement performance.

The variability in delivery performance among suppliers stems from factors such as transportation logistics, inventory management practices, and order complexities. Sodhi and Tang (2012) argue that suppliers' ability to meet delivery commitments fluctuates due to transit times, production schedules, and inventory availability uncertainties resulting in different values of responses for a single input/output in a single DMU at different time points. This variability can manifest in several ways, such as differing lead times for the same product across different orders, inconsistencies in meeting delivery deadlines, and varying reliability in the quality of delivered goods. These fluctuations pose quite a challenge to inventory management, production planning, and customer service as firms have to constantly vary their operations while dealing with uncertain delivery schedules. For instance, a supplier may reliably manage to meet their delivery schedules to one client but not another because the order sizes are different, the shipping routes are different, or priority has been accorded to different clients.

For example, some component in the automotive industry may be received from different suppliers by a manufacturer. All the suppliers' performances vary because of differences in production capacity, geographical location, and efficiency of logistic management. One may deliver high-quality components on time but at higher prices than others, while another has better prices but less reliable delivery schedules. These multiple factors must be evaluated to optimize the supply chain for cost, quality, and reliability. Also, a single supplier can have different components in quality and price at different times, leading to multi-scenario responses for data analysis. In such situations, which data to consider for supply evaluation matters. Should we take the average of the responses, the maximum, the minimum, or the mode is questionable?

In this multi-scenario context, DEA can be employed to assess the efficiency of each supplier by considering each scenario separately. This method provides a more granular supplier performance analysis, highlighting strengths and weaknesses across different conditions. For instance, DEA can be used to create efficiency frontiers for each scenario, allowing the company to understand how each supplier performs under varying conditions and to identify which suppliers consistently perform well across multiple scenarios (Li et al., 2019).

Thus, addressing the variability in supplier performance requires a multi-scenario analysis approach. This approach evaluates multiple potential outcomes or responses for a single input /output from a single DMU under different conditions, allowing firms to anticipate and manage uncertainties better. By doing so, the firm can develop more robust supplier selection strategies and procurement, inventory management, and production planning strategies.

Thus multi-scenario analysis enables firms to prepare for various kinds of disruptions and come up with contingency plans that are important in building the capability of maintaining operations under adverse conditions. Collaborative relationships with suppliers and techniques of risk management, like dual-sourcing, safety stock, and flexible contracts, become critical in increasing the ability of supply chains to withstand variability.

As mentioned above, DEA models typically handle precise data and do not account for uncertain input and output situations. Many scholars have explored these scenarios, considering fuzzy data, interval data, probabilistic data, and other forms of uncertainty in datasets. They have proposed various approaches to address this, including mean value and variance models, robust DEA, multiple criteria decision-making (MCDM) models, and several other techniques.

A study by Zahedi-Seresht et al. (2021) addresses cases where multiple alternative scenarios represent uncertainty in the dataset. The initial model for problems featuring various alternative scenarios in their inputs and outputs is developed directly from the definition of the relative efficiency formula, akin to those used in traditional DEA models. Their model was illustrated using a numerical example with 10 DMUs and three scenarios for input and output values as depicted in Table 6. They considered single scenarios for inputs which are manpower and Assets of a engineering company, but multiple scenarios for Output.

Table 6 – Inputs and Outputs for different scenarios

	Manpower	Assets	Output - Payments		
	Input 1	Input 2	Scenario 1	Scenario 2	Scenario 3
DMU_1	606	293	3054	2974	2455
DMU_2	797	569	897	948	862
DMU_3	247	614	777	836	760
DMU_4	376	126	987	829	860
DMU_5	876	553	644	670	3852
DMU_6	2766	365	2814	2999	2360
DMU_7	245	715	2305	2009	2196
DMU_8	145	147	128	109	127
DMU_9	136	14,054	5070	5370	5828
DMU_{10}	141	1559	3000	2900	2756

The key contribution of their paper is the development of models to assess efficiency, considering the existence of multiple alternative scenarios, each with a specified probability of occurring. Initially, the paper introduces a model based on the traditional CCR framework. This model considers an aggregated Decision Making Unit (DMU), which is defined as the weighted sum of all possible alternative scenarios.

Also, there can be DMUs with different internal structures. The two-stage DEA model is a unique version designed to assess the efficiency of Decision-Making Units (DMUs) with a two-step internal process. In this model, initial inputs are first converted into intermediate outputs. These intermediate outputs are transformed into the final outputs in the subsequent stage. A study by M. Alimohammadi Ardekani (2016) emphasizes that traditional Data Envelopment Analysis (DEA) models assess efficiency based solely on inputs and outputs, without considering the internal structure of Decision-Making Units (DMUs). Consequently, these conventional models cannot pinpoint the specific reasons and origins of inefficiency within the DMUs.

A two-stage DEA model is a specific type of network DEA model designed to measure the relative efficiency of Decision Making Units (DMUs) with a two-stage internal structure. In this model, the initial inputs are first transformed into intermediate outputs, which are then converted into final outputs in the subsequent sub-process. The two-stage DEA model allows for the decomposition of the overall efficiency of DMUs into the efficiencies of each individual stage. The authors propose a general method for estimating network efficiency, illustrated through a case study of the Regional Electricity Company of Iran. The two stages of the DMU are the Production stage and the Distribution stage. The following is sample data that is used in their Analysis.

Table 7- The results of two-stage DEA models for Iranian Electricity Company

DMU	P. process	Rank	D. process	Rank	O. process	Rank
DMU1	0.455	9	0.456	11	0.208	14
DMU2	1.000	1	1.000	1	1.000	1
DMU3	0.848	5	1.000	1	0.848	3
DMU4	1.000	1	0.506	9	0.506	11
DMU5	0.534	8	0.505	10	0.269	13
DMU6	0.774	7	1.000	1	0.774	5
DMU7	1.000	1	0.569	8	0.569	9
DMU8	0.955	3	0.392	12	0.374	12
DMU9	1.000	1	0.693	6	0.693	7
DMU10	0.889	4	0.637	7	0.567	10
DMU11	1.000	1	0.945	2	0.945	2
DMU12	1.000	1	0.759	5	0.759	6
DMU13	0.775	6	0.838	4	0.649	8
DMU14	0.973	2	0.863	3	0.840	4
Mean	0.872	-	0.726	-	0.643	-

As per table 7, DMUs 2, 4, 7, 9, 11, and 12 have excelled in the production stage, while DMU1 has performed the worst. In the distribution stage, DMUs 2, 3, and 6 have achieved top ranks, with DMU8 performing the worst. Despite their top ranks in production, DMUs 4 and 7 did not reach the efficiency frontier in the distribution stage, placing them in an unfavorable position overall. Further, They emphasize that, even though the traditional two-stage DEA model assumes that the input/output and intermediate output vectors are free from any uncertainty, real-world scenarios often involve data that is noisy or subject to perturbations, making it necessary to develop a two-stage DEA model that can handle such variations. Robust optimization (RO) has emerged as a popular method for addressing data uncertainty. While stochastic programming (SP) has traditionally been used to manage uncertainty, their paper utilizes the RO which is used as an alternative for performing sensitivity analysis and addressing the challenges of stochastic programming. Table 8 shows the results of the robust two stage model where the perturbations are set to 0.05 and 0.10, respectively.

Table 8- The results of robust two-stage DEA models for Iranian Electricity company

DMU	Robust Two-Stage DEA Model(e=0.05)						Robust Two-Stage DEA Model (e=0.1)					
	Production process	Rank	Distribution process	Rank	Overall process	Rank	Production process	Rank	Distribution process	Rank	Overall process	Rank
DMU1	0.433	8	0.434	11	0.188	14	0.414	9	0.415	11	0.171	14
DMU2	0.952	1	0.952	1	0.907	1	0.909	1	0.909	1	0.826	1
DMU3	0.803	4	0.952	1	0.764	3	0.763	5	0.909	1	0.693	3
DMU4	0.952	1	0.482	9	0.459	11	0.909	1	0.460	9	0.418	11
DMU5	0.508	7	0.480	10	0.244	13	0.485	8	0.459	10	0.223	13
DMU6	0.735	5	0.952	1	0.700	5	0.700	6	0.909	1	0.636	5
DMU7	0.952	1	0.542	8	0.516	9	0.909	1	0.517	8	0.470	9
DMU8	0.908	2	0.373	12	0.339	12	0.867	3	0.302	12	0.262	12
DMU9	0.952	1	0.660	6	0.629	7	0.909	1	0.630	6	0.573	7
DMU10	0.843	3	0.607	7	0.512	10	0.805	4	0.580	7	0.467	10
DMU11	0.952	1	0.900	2	0.857	2	0.909	1	0.859	2	0.781	2
DMU12	0.952	1	0.723	5	0.689	6	0.909	1	0.690	5	0.627	6
DMU13	0.734	6	0.798	4	0.586	8	0.698	7	0.762	4	0.531	8
DMU14	0.926	1	0.822	3	0.762	4	0.884	2	0.780	3	0.690	4
Mean	0.829	-	0.691	-	0.582	-	0.791	-	0.656	-	0.526	-

The findings show that incorporating perturbations into the data led to changes in the efficiency rankings of decision-making units. This underscores the significant influence that data uncertainty can have on these rankings.

Data of Interest

Overview for Data

Canada has one of the most sophisticated and innovative indoor agriculture industries in the world, with global market successes. The greenhouse vegetable and mushroom industries are substantive parts of Canadian agriculture and a major economic driver for Canada as a whole. According to the Statistical overview of the Canadian greenhouse vegetable and mushroom industry, in 2023, these two sectors saw $2.2 billion in farm gate sales in 2022, thereby underlining the economic importance of the businesses in question. The export value also reached approximately $1.9 billion, which clearly underlined Canada's leading position in the world markets. Of the large number of crops produced in Canadian greenhouses, tomatoes, peppers, and cucumbers hold top places. In 2022, these three vegetables represented 96% of the total production of greenhouse vegetables. This significant concentration illustrates the specialized focus and expertise within the industry, ensuring high-quality produce that meets domestic and international demand. The continued successes and growth are thus a reflection of the commitment to ensure that Canada remains competitive based on insemination of technology and agricultural innovation

Canadian greenhouse growers have been making huge investments in modern greenhouses to raise volumes and answer demands for fresh vegetables throughout the year. Nowadays, a significant portion of greenhouse facilities across Canada is highly automated. This trend reflects a widespread commitment among growers to adopt the latest technological advancements continually. By doing so, they aim to enhance production efficiency, minimize labor costs and inputs, and improve the quality of their products. The drive towards embracing innovative technologies has empowered farmers to extend the natural harvest seasons, thereby achieving continuous gains in efficiency. These improvements have significantly boosted the competitiveness of Canadian greenhouse growers and led to noticeable enhancements in product quality.

The Canadian vegetable greenhouse industry is strategically concentrated near the southern border with the United States. Notable regions include the Fraser Valley in British Columbia, Southern Ontario, and Quebec. These locations offer several advantages: they are close to major consumer markets, have access to efficient transportation networks, and provide favorable growing conditions. This strategic positioning not only supports the logistics of distributing fresh produce but also optimizes growing conditions, contributing to the overall success and sustainability of the industry.

Recent Statistics

There were 934 commercial greenhouse vegetable operations in Canada in 2022. These produced a remarkable 752,685 metric tons of vegetables. This indicates an increase of 5% in the number of operations and 7% in production from 2021, the year prior. Ontario continued to dominate the industry regarding the production of greenhouse vegetables, contributing about 71% of the country's total output. British Columbia was next in contribution at 14.5%, followed by Quebec at 8%. The remaining 6.5% of greenhouse vegetable production was distributed among the other provinces. The data indicates the continuing growth of Canada's greenhouse vegetable industry and regional distribution.

These producers cultivate a wide variety of crops, including tomatoes, cucumbers, lettuce, bell peppers, green and yellow beans, eggplants, strawberries, various herbs, and microgreen vegetables. However, tomatoes, cucumbers, and bell peppers are the main crops grown by the majority of producers; these are the three major greenhouse vegetable crops produced in Canada.

Tomatoes occupy the largest share and contribute 36% to sales in the greenhouse vegetable sector. The next in line are cucumbers, with 33%, and finally, bell peppers with 25% of the sales. All three of these crops together build the foundation of Canada's greenhouse vegetable market, defining their role for the entire farmed produce.

The economic impact of this greenhouse production of fruits and vegetables has been high. In 2022, total sales in that sector were up 11 percent over the previous year to a record $2.2 billion. To further put this change into perspective, these increases are part of a longer trend of yearly rises beginning all the way back in 2013, which speaks to the solid and continued growth and escalating importance of greenhouse agriculture to Canada's economy. This steady increase is a pointer to increased demand for greenhouse-grown produce, coupled with better technologies and farming practices that achieve productivity and efficiency gains in the industry.

In 2022, the value of greenhouse vegetable exports from Canada increased significantly from 2021 by 7.5 percent to over $1.4 billion, making it the most valuable category of all fresh produce exports in the country, including fruits, mushrooms, field vegetables, and potatoes, and accounting for 39% of the total value of all fresh produce exports. Cucumbers, peppers, and tomatoes dominated the greenhouse vegetable production market. Each of these types of vegetables held quite a share of export value: cucumbers and peppers had 34%, while tomatoes had 32%.

Mushroom production, like greenhouse vegetables, occurs in a controlled environment and is a year-round activity. In this dark, specially climatized setting, growing conditions are optimized by controlling temperature and humidity. Mushroom growing is a continuous process with 24 hours a day, 7 days a week harvesting. Canadian mushroom growers produced 139,090 metric tonnes of mushrooms in 2022, up 0.9 percent from the previous year. The gain was supported by a 1.8 percent increase in cultivated area and 7.2 percent increase in harvested area.

Improvement to the financial performance of the mushroom industry occurred in 2022. The total value of mushrooms sold leaped by 6.3 percent to $694.5 million as fresh and processed prices went up. Ontario and British Columbia continue to be Canada's leading mushroom-producing provinces, with the two accounting for about 93 percent of the total value of Canadian mushrooms. Ontario alone accounted for over half of the country's volume.

Regarding exports, the United States continued to be the largest market for Canadian mushrooms, much like the trend for greenhouse vegetable exports. Over 97% of the value of fresh mushroom exports was destined for the U.S. Over the past five years, the value of mushroom exports increased by 66% to USD 446.0 million in 2022. This clearly indicates the rise in demand and market value for Canadian mushrooms in the global market, particularly in the United States.

COVID vs Greenhouse Revenue

Food security is becoming of critical importance to society, driven by the challenge of feeding a global population that is projected to be 9.6 billion by the year 2050. This is further complicated by the continued decrease in the availability of arable land. Recent disruptions to international trade, largely a product of responses to the COVID-19 pandemic, have again brought into focus the need for countries to be self-sufficient in food production.

In the case of Canada, the country is nearly self-sufficient in terms of most foodstuffs specified—including, of course, meat and dairy. At the same time, in terms of fresh vegetables, it relies on international imports. This dependency on global supply chains really exposes Canadian food security to several international trade disruption scenarios.

One of the most promising solutions to this vulnerability is in the expansion of the greenhouse vegetable industry. All year round, greenhouses can credibly produce fresh vegetables, independent of climatic conditions outside. With advanced agricultural technologies, greenhouses can stake a claim to provide fresh produce on a stable and continuous basis. This move toward localizing food production will significantly increase Canada's self-sufficiency in food and minimize the risks associated with reliance upon international imports.

This could add further to the benefits accruing from the development of the greenhouse industry in terms of the number of jobs it could create within the agricultural value chain, stimulating the economy while helping the environment with a reduced carbon footprint from the transportation of food over long distances. Therefore, by investing and expanding its greenhouse vegetable production, Canada can take giant steps toward ensuring that it has a food supply resilient to the next global shocks.

The COVID-19 pandemic has evidenced how the application of adapted supplier selection criteria will go a long way in ensuring supply chain resilience. Given changes that were happening in consumer behavior and disruptions to supply chains, many operators in Canada had to adapt rapidly during the outbreak of COVID-19.

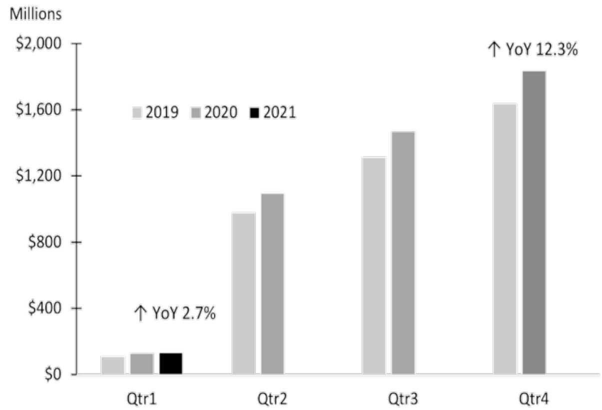
This adaptability included selecting suppliers who can ensure timely and reliable deliveries despite global transport challenges. With that, the selection of suppliers has moved further towards local suppliers to drive down risks in international shipping delays and border restrictions. This move also impinged directly on the efficiency indicators, where the dependency on local suppliers reduced lead times and transportation costs, improving overall supply chain efficiency.

Coming off the challenges of 2020, there is an obvious question for the Canadian greenhouse sector. Will 2021, declared by the United Nations to be the International Year of Fruits and Vegetables, be as successful? Based on Farm Credit Canada, greenhouse sales in 2020 showed a 9.4% year-over-year increase, thanks to a good fruit and vegetable crop. A number of challenges, though, may keep this growth from carrying over into 2021. Rising operational costs, plus appreciation of the Canadian dollar, may prove to be the most significant challenge to producers.

In a year ruled by the COVID-19 pandemic, greenhouse-grown fresh vegetables have notched up their fastest growth in receipts since 2012. For the eighth year in a row, greenhouse-grown fresh vegetables recorded a 12.3% jump to $1.8 billion in 2020 (as seen in Figure 1). This spectacular growth was off the back of a solid performance in 2019, where sales grew 5.0% to end at $1.6 billion.

As soon as the pandemic arrived, operators in greenhouses had to adapt to new ways of consumer behavior such as curbside pickup and online shopping, boosting farm cash receipts by an astonishing 19.8% year-over-year in the first quarter of 2020. That continued into second and third quarters, all suggesting that one key factor to sustaining growth was quick adaptation in the sector to the challenges posed by the pandemic. These changes underscored the sector's agility in response to the pandemic, emphasizing the importance of having a flexible supply chain and efficient supplier selection processes.

Figure 1 – Canadian Greenhouse fresh vegetable cash receipts



Source- [Despite remarkable greenhouse sales in 2020 – uncertainty lies in the year ahead | FCC. (n.d.). https://www.fcc-fac.ca/en/knowledge/economics/greenhouse-sales](https://www.fcc-fac.ca/en/knowledge/economics/greenhouse-sales)

Within the period of 2015-2020, greenhouse areas allocated for fruit and vegetable production increased drastically by 23.9%. Last year, it rose to a total capacity of 1,809 hectares. This growth was reflected in the increased production of peppers and cucumbers, which are two of the three primary fresh vegetables produced in large quantities. In contrast, tomato production, despite being the largest in terms of both value and volume, experienced a slight decline of 3.2%.

Nevertheless, tomatoes continued to lead sales growth, pulling off an impressive 12.1% increase. Next came cucumbers with a sales rise of 9.4%, followed by peppers with 7.3%. The trio of tomato, cucumber, and pepper sales accounted for 92.5%, thereby giving strong support to their leading position in the market.

The bottom line is that the COVID-19 pandemic accelerated the need for a supply chain in the agriculture sector to be both efficient and resilient. Notably, it concerns greenhouse vegetable production. Investment in local suppliers and more efficient supply chain practices would accordingly enhance Canada's food security by lessening its reliance on international imports and saving up a resilient supply of food for whatever global shock might come.

Inputs and Outputs for DEA

To illustrate the proposed mathematical model for multi-scenario Data Envelopment Analysis (DEA), we will use data from greenhouse, sod, and nursery operations across Canada, sourced from Statistics Canada in 2024. This model focuses on evaluating the efficiency of these operations over multiple years. The output variable in our analysis is the total value of greenhouse products produced. The input variables comprise various operating expenses incurred by specialized greenhouse producers. These expenses are categorized into several distinct types, which we will treat as separate inputs in the DEA model. Specifically, we consider expenses related to specialized greenhouse vegetables and specialized greenhouse flowers and plants.

Our multi-scenario approach involves analyzing data collected over five years: 2019, 2020, 2021, 2022, and 2023. This longitudinal dataset allows us to assess trends and changes in efficiency over time. The data serves a crucial role for federal and provincial agricultural departments, as well as producer associations, by aiding in market trend analysis and examining domestic production with a particular focus on import dynamics. Additionally, this survey contributes to the agricultural receipts program of Statistics Canada, ultimately feeding into the System of National Accounts.

Agriculture and Agri-Food Canada, along with other federal departments, utilize this information to develop and administer agricultural policies. Provincial departments leverage the data for production and price analysis, as well as for economic research. The survey is administered under the Integrated Business Statistics Program (IBSP), which consolidates approximately 200 individual business surveys into a cohesive master survey program. The IBSP aims to collect detailed industry and product information at the provincial level while minimizing redundancy across different survey questionnaires.

For the purpose of this analysis, we will focus on three key provinces: British Columbia, Ontario, and Quebec. These provinces will serve as the three decision-making units (DMUs) in our DEA model. Within each province, farms are classified into one of four categories based on their involvement in greenhouse, sod, and nursery activities, as well as their primary North American Industry Classification System (NAICS) code. The four categories are:

1. Greenhouse - Floriculture
2. Greenhouse - Other
3. Sod
4. Nursery

Farms that produce multiple commodities are assigned to a category based on the predominant contribution of each commodity at the provincial level. The combination of these categories and provinces forms what is known as the sampling cell for our analysis.

The target population for this study includes all entities operating greenhouse, sod, or nursery operations that meet Statistics Canada's definition of a farm. A farm is defined as an operation that produces at least one agricultural product and reports revenue and/or expenses related to that production to the Canada Revenue Agency. Certain small populations are excluded from the target population of the Annual Greenhouse, Sod, and Nursery Survey. These exclusions include farms in Canada's three territories, institutional farms, community farms, greenhouses that produce marijuana, and greenhouses or nurseries that produce tree seedlings for reforestation.

Input - Specialized greenhouse producers' operating expenses

There are primarily two categories of greenhouse producers: those that specialize in greenhouse vegetables and those that specialize in greenhouse flowers and plants. These categories exclude mixed operations (involving both vegetables and flowers/plants) and cannabis operations. The total expenses for each category are determined by summing several specific types of costs.

For greenhouse vegetable producers, the total expenses encompass the following:

- **Plant Material Purchases for Growing On**: This includes the cost of acquiring flowers, plants, cuttings, seedlings, seeds, and bulbs that are intended to be grown on-site. The value is calculated before any sales tax is applied.
- **Plant Material Purchases for Resale**: This covers the expense of buying flowers, plants, cuttings, seedlings, seeds, and bulbs for the purpose of resale, also before sales tax.
- **Gross Yearly Payroll**: This accounts for the total wages paid to both seasonal and permanent laborers employed by the greenhouse operation.
- **Electricity Costs**: These are the expenses related to electricity usage for various purposes such as lighting, running airflow fans, and heating within the greenhouse.
- **Fuel Costs**: This includes all fuel-related expenses incurred for the operation of the greenhouse.
- **Other Crop Expenses**: This category encompasses a wide range of costs associated with crop production, including fertilizers, pesticides, pollination services, irrigation, containers, packaging, bioprograms, and growing mediums like soil, peat moss, vermiculite, perlite, sand, Styrofoam, and sawdust.

- **Other Operating Expenses**: These are additional expenses necessary for running the greenhouse operation. They include interest payments, land taxes, insurance premiums, advertising costs, repairs to farm buildings, machinery, agricultural equipment, and vehicles, costs of contract work, and expenses related to telephone and telecommunications services.

Similarly, for greenhouse flower and plant producers, the total expenses include the same categories as listed above:

- **Plant Material Purchases for Growing On**: The value of flowers, plants, cuttings, seedlings, seeds, and bulbs purchased for cultivation, calculated before sales tax.
- **Plant Material Purchases for Resale**: The cost of flowers, plants, cuttings, seedlings, seeds, and bulbs bought for resale, also before sales tax.
- **Gross Yearly Payroll**: The total wages paid to both seasonal and permanent employees.
- **Electricity Costs**: Expenses for electricity used for lighting, airflow fans, and heating.
- **Fuel Costs**: All fuel-related expenses for the operation of the greenhouse.
- **Other Crop Expenses**: Costs related to fertilizers, pesticides, pollination, irrigation, containers, packaging, bioprograms, and growing mediums such as soil, peat moss, vermiculite, perlite, sand, Styrofoam, and sawdust.
- **Other Operating Expenses**: Additional operational costs including interest payments, land taxes, insurance, advertising, repairs to buildings, machinery, agricultural equipment, and vehicles, contract work, and telephone and telecommunications services.

By considering all these specific types of expenses, we can accurately determine the total expenses for both specialized greenhouse vegetable and specialized greenhouse flower and plant operations.

Table 9 – Greenhouse Producers Operating Expenses in British Colombia (in Dollars)

Producers	Geography	British Colombia				
	Expenses	**2019**	**2020**	**2021**	**2022**	**2023**
Specialized greenhouse vegetable	Plant material purchases for growing on	21,519,817	27,276,523	23,317,772	22,990,864	20,606,968
	Plant material purchases for resale	0	0	0	0	0
	Gross yearly payroll	67,227,559	73,003,694	79,709,266	80,127,442	80,431,798
	Electricity	5,240,308	5,984,723	6,099,920	6,871,855	6,228,145
	Fuel	29,186,819	29,363,914	28,435,349	32,494,227	32,708,966
	Other crop expenses	46,177,481	60,015,564	59,962,203	63,810,630	64,695,604
	Other operating expenses	60,823,849	53,045,269	54,839,394	57,937,017	58,009,193
	Total operating expenses	***230,175,833***	***248,689,687***	***252,363,903***	***264,232,035***	***262,680,674***
Specialized greenhouse flower and plant	Plant material purchases for growing on	39,412,071	48,923,400	53,778,894	55,817,591	52,798,755
	Plant material purchases for resale	11,500,828	11,982,077	17,598,720	17,793,794	15,406,573
	Gross yearly payroll	55,038,249	65,754,148	71,524,705	72,593,871	77,511,029
	Electricity	5,866,190	7,096,710	6,806,997	7,462,695	7,593,961
	Fuel	8,491,207	9,639,198	10,582,885	11,431,840	12,857,194
	Other crop expenses	28,105,213	30,682,080	33,808,409	34,812,582	35,439,222
	Other operating expenses	32,829,159	28,883,018	32,467,212	33,094,951	39,058,410
	Total operating expenses	***181,242,917***	***202,960,631***	***226,567,822***	***233,007,324***	***240,665,144***

Source- *Statistics Canada. Table 32-10-0025-01 Specialized greenhouse producers' operating expenses*

Table 10 - Greenhouse Producers Operating Expenses in Ontario (in Dollars)

Producers	Geography	Ontario				
	Expenses	**2019**	**2020**	**2021**	**2022**	**2023**
Specialized greenhouse vegetable	Plant material purchases for growing on	96,398,673	107,006,294	122,041,230	132,697,073	137,179,630
	Plant material purchases for resale	0	0	0	0	0
	Gross yearly payroll	258,822,641	289,405,268	354,291,694	377,127,111	389,508,955
	Electricity	22,236,382	26,221,262	36,253,539	37,494,931	43,494,120
	Fuel	99,888,431	103,064,533	115,796,766	120,708,555	129,462,119
	Other crop expenses	167,517,248	180,527,603	207,903,337	247,370,590	269,598,142
	Other operating expenses	209,826,056	260,323,454	268,258,835	292,804,007	330,275,890
	Total operating expenses	***854,689,431***	***966,548,414***	***1,104,545,402***	***1,208,202,267***	***1,299,518,856***
Specialized greenhouse flower and plant	Plant material purchases for growing on	116,984,393	117,114,919	124,736,489	128,424,019	131,377,771
	Plant material purchases for resale	78,565,367	79,058,179	58,086,222	60,078,075	69,239,946
	Gross yearly payroll	165,233,241	158,110,005	156,074,932	159,735,990	182,702,743
	Electricity	13,736,008	14,011,733	10,146,487	12,472,842	14,976,041
	Fuel	29,440,026	22,218,312	23,458,257	25,172,680	30,652,589
	Other crop expenses	90,533,964	91,949,279	98,325,549	126,304,382	132,312,398
	Other operating expenses	113,517,481	126,025,519	132,308,286	137,429,016	150,329,879
	Total operating expenses	***608,010,480***	***608,487,946***	***603,136,222***	***649,617,004***	***711,591,369***

Source- *Statistics Canada. Table 32-10-0025-01 Specialized greenhouse producers' operating expenses*

Table 11 - Greenhouse Producers Operating Expenses in Quebec (in Dollars)

Producers	Geography	Quebec				
	Expenses	2019	2020	2021	2022	2023
Specialized greenhouse vegetable	Plant material purchases for growing on	6,017,566	6,455,788	7,074,880	7,990,710	7,959,193
	Plant material purchases for resale	0	0	0	0	0
	Gross yearly payroll	30,909,289	38,585,698	49,847,384	51,268,987	51,750,570
	Electricity	7,471,815	7,699,720	11,586,660	11,867,188	12,055,220
	Fuel	9,211,085	8,469,340	10,131,665	10,628,727	11,344,359
	Other crop expenses	16,953,893	19,183,207	27,314,033	27,937,367	24,936,979
	Other operating expenses	19,327,580	22,762,635	31,169,028	31,847,033	24,705,908
	Total operating expenses	***89,891,228***	***103,156,388***	***137,123,650***	***141,540,012***	***132,752,229***
Specialized greenhouse flower and plant	Plant material purchases for growing on	25,969,415	25,706,469	17,413,924	22,411,362	23,252,384
	Plant material purchases for resale	7,294,212	9,328,448	12,965,656	13,096,244	12,108,571
	Gross yearly payroll	38,240,437	36,869,261	39,385,096	39,962,938	45,693,275
	Electricity	2,791,885	2,239,668	2,072,583	2,197,625	2,747,157
	Fuel	10,554,802	9,406,911	7,793,968	8,002,060	9,314,974
	Other crop expenses	17,541,982	19,135,521	18,233,712	21,152,594	21,878,127
	Other operating expenses	21,237,890	20,408,729	21,789,628	22,534,052	23,180,953
	Total operating expenses	***123,630,623***	***123,095,007***	***119,654,567***	***129,356,875***	***138,175,442***

Source- *Statistics Canada. Table 32-10-0025-01 Specialized greenhouse producers' operating expenses*

Output- Total value of greenhouse products

The output considered in the study is the Total value of sales, which comprises two main components: Fruit and vegetable sales, and Flower and plant sales and resales.

1. **Fruit and vegetable sales**: This category represents the value of produce sold directly from farms, excluding taxes. It encompasses a variety of items, including greenhouse Chinese vegetables, herbs, and sprouts.
2. **Flower and plant sales and resales**:
   - **Flower and plant sales**: The value of flowers and plants sold directly from farms, excluding sales tax. Notably, this category does not include sales related to cannabis.
   - **Flower and plant resales**: This include the resale value of flowers and plants, representing transactions where previously sold items are sold again.

In summary, the study analyzes the overall sales revenue generated from these agricultural sectors, distinguishing between direct sales of fruits, vegetables, flowers, and plants from farms, and subsequent resale activities within the flower and plant market.

Table 12- consists of the Output data components for British Colombia (in Dollars)

Geography	**British Columbia**				
Scenario	**2019**	**2020**	**2021**	**2022**	**2023**
Fruit and vegetable sales	306,335,083	329,623,922	322,817,515	349,544,705	369,109,987
Flower and plant sales and resales	342,733,310	400,565,796	452,481,666	487,795,340	514,430,769
Flower and plant sales	316,254,990	365,706,725	410,072,083	440,756,031	455,614,010
Flower and plant resales	26,478,320	34,859,071	42,409,583	47,039,309	58,816,759
Total greenhouse sales	***649,068,393***	***730,189,718***	***775,299,181***	***837,340,045***	***883,540,756***

Source- *Statistics Canada. Table 32-10-0023-01 Total value of greenhouse products*

Table 13- Output data components for Ontario (in Dollars)

Geography	**Ontario**				
Scenario	**2019**	**2020**	**2021**	**2022**	**2023**
Fruit and vegetable sales	1,067,387,947	1,190,662,602	1,300,512,754	1,509,966,196	1,667,977,048
Flower and plant sales and resales	817,586,186	839,461,539	913,740,368	1,007,274,383	1,047,366,136
Flower and plant sales	705,592,622	724,641,118	804,652,953	894,655,878	924,105,092
Flower and plant resales	111,993,564	114,820,421	109,087,415	112,618,505	123,261,044
Total greenhouse sales	***1,884,974,133***	***2,030,124,141***	***2,214,253,122***	***2,517,240,579***	***2,715,343,184***

Source- *Statistics Canada. Table 32-10-0023-01 Total value of greenhouse products*

Table 14 - Output data components for Quebec (in Dollars)

Geography	**Quebec**				
Scenario	**2019**	**2020**	**2021**	**2022**	**2023**
Fruit and vegetable sales	146,811,420	174,061,503	211,307,863	248,311,808	269,349,188
Flower and plant sales and resales	215,194,730	240,733,354	267,282,090	271,714,624	273,384,629
Flower and plant sales	193,479,561	215,999,546	238,621,177	245,331,325	248,375,888
Flower and plant resales	21,715,169	24,733,808	28,660,913	26,383,299	25,008,741
Total greenhouse sales	***362,006,150***	***414,794,857***	***478,589,953***	***520,026,432***	***542,733,817***

Source- *Statistics Canada. Table 32-10-0023-01 Total value of greenhouse products*

The following tables consist of the summarized data, including inputs and outputs for each scenario and each DMU collectively.

Table 15- Input data for all the DMU's

DMU	**Input 1**					**Input 2**					**Output**				
Scenarios	***2019***	***2020***	***2021***	***2022***	***2023***	***2019***	***2020***	***2021***	***2022***	***2023***	***2019***	***2020***	***2021***	***2022***	***2023***
BC	230,175,833	248,689,687	252,363,903	264,232,035	262,680,674	181,242,917	202,960,631	226,567,822	233,007,324	240,665,144	649,068,393	730,189,718	775,299,181	837,340,045	883,540,756
Ontario	854,689,431	966,548,414	1,104,545,402	1,208,202,267	1,299,518,856	608,010,480	608,487,946	603,136,222	649,617,004	711,591,369	1,884,974,133	2,030,124,141	2,214,253,122	2,517,240,579	2,715,343,184
Quebec	89,891,228	103,156,388	137,123,650	141,540,012	132,752,229	123,630,623	123,095,007	119,654,567	129,356,875	138,175,442	362,006,150	414,794,857	478,589,953	520,026,432	542,733,817

Developing the Mathematical Model for each DMU

DEA model for BC 2019

$Max\ \theta = 649{,}068{,}393u_1$
Subject to

$$649{,}068{,}393u_1 - 230{,}175{,}833v_1 - 181{,}242{,}917v_2 \leq 0$$
$$1{,}884{,}974{,}133u_1 - 854{,}689{,}431v_1 - 608{,}010{,}480v_2 \leq 0$$
$$362{,}006{,}150u_1 - 89{,}891{,}228v_1 - 123{,}630{,}623v_2 \leq 0$$
$$230{,}175{,}833v_1 + 181{,}242{,}917v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for BC 2020

$Max\ \theta = 730{,}189{,}718u_1$
Subject to

$$730{,}189{,}718u_1 - 248{,}689{,}687v_1 - 202{,}960{,}631v_2 \leq 0$$
$$2{,}030{,}124{,}141u_1 - 966{,}548{,}414v_1 - 608{,}487{,}946v_2 \leq 0$$
$$414{,}794{,}857u_1 - 103{,}156{,}388v_1 - 123{,}095{,}007v_2 \leq 0$$

$$248{,}689{,}687v_1 + 202{,}960{,}631v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for BC 2021

$Max\ \theta = 775{,}299{,}181u_1$

Subject to

$$775{,}299{,}181u_1 - 252{,}363{,}903v_1 - 226{,}567{,}822v_2 \leq 0$$
$$2{,}214{,}253{,}122u_1 - 1{,}104{,}545{,}402v_1 - 603{,}136{,}222v_2 \leq 0$$
$$478{,}589{,}953u_1 - 137{,}123{,}650v_1 - 119{,}654{,}567v_2 \leq 0$$
$$252{,}363{,}903v_1 + 226{,}567{,}822v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for BC 2022

$Max\ \theta = 837{,}340{,}045u_1$

Subject to

$$837{,}340{,}045u_1 - 264{,}232{,}035v_1 - 233{,}007{,}324v_2 \leq 0$$
$$2{,}517{,}240{,}579u_1 - 1{,}208{,}202{,}267v_1 - 649{,}617{,}004v_2 \leq 0$$
$$520{,}026{,}432u_1 - 141{,}540{,}012v_1 - 129{,}356{,}875v_2 \leq 0$$
$$264{,}232{,}035v_1 + 233{,}007{,}324v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for BC 2023

$Max\ \theta = 883{,}540{,}756u_1$

Subject to

$$883{,}540{,}756u_1 - 262{,}680{,}674v_1 - 240{,}665{,}144v_2 \leq 0$$
$$2{,}715{,}343{,}184u_1 - 1{,}299{,}518{,}856v_1 - 711{,}591{,}369v_2 \leq 0$$
$$542{,}733{,}817u_1 - 132{,}752{,}229v_1 - 138{,}175{,}442v_2 \leq 0$$
$$262{,}680{,}674v_1 + 240{,}665{,}144v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Ontario 2019

$Max\ \theta = 1{,}884{,}974{,}133u_1$

Subject to

$$649{,}068{,}393u_1 - 230{,}175{,}833v_1 - 181{,}242{,}917v_2 \leq 0$$
$$1{,}884{,}974{,}133u_1 - 854{,}689{,}431v_1 - 608{,}010{,}480v_2 \leq 0$$
$$362{,}006{,}150u_1 - 89{,}891{,}228v_1 - 123{,}630{,}623v_2 \leq 0$$
$$854{,}689{,}431v_1 + 608{,}010{,}480v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Ontario 2020

$Max\ \theta = 2{,}030{,}124{,}141u_1$

Subject to

$$730{,}189{,}718u_1 - 248{,}689{,}687v_1 - 202{,}960{,}631v_2 \leq 0$$
$$2{,}030{,}124{,}141u_1 - 966{,}548{,}414v_1 - 608{,}487{,}946v_2 \leq 0$$
$$414{,}794{,}857u_1 - 103{,}156{,}388v_1 - 123{,}095{,}007v_2 \leq 0$$
$$966{,}548{,}414v_1 + 608{,}487{,}946v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Ontario 2021

$Max\ \theta = 2{,}214{,}253{,}122u_1$

Subject to

$$775{,}299{,}181u_1 - 252{,}363{,}903v_1 - 226{,}567{,}822v_2 \leq 0$$
$$2{,}214{,}253{,}122u_1 - 1{,}104{,}545{,}402v_1 - 603{,}136{,}222v_2 \leq 0$$
$$478{,}589{,}953u_1 - 137{,}123{,}650v_1 - 119{,}654{,}567v_2 \leq 0$$
$$1{,}104{,}545{,}402v_1 + 603{,}136{,}222v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Ontario 2022

$Max\ \theta = 2{,}517{,}240{,}579u_1$

Subject to

$$837{,}340{,}045u_1 - 264{,}232{,}035v_1 - 233{,}007{,}324v_2 \leq 0$$
$$2{,}517{,}240{,}579u_1 - 1{,}208{,}202{,}267v_1 - 649{,}617{,}004v_2 \leq 0$$
$$520{,}026{,}432u_1 - 141{,}540{,}012v_1 - 129{,}356{,}875v_2 \leq 0$$
$$1{,}208{,}202{,}267v_1 + 649{,}617{,}004v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Ontario 2023

$Max\ \theta = 2{,}715{,}343{,}184u_1$

Subject to

$$883{,}540{,}756u_1 - 262{,}680{,}674v_1 - 240{,}665{,}144v_2 \leq 0$$
$$2{,}715{,}343{,}184u_1 - 1{,}299{,}518{,}856v_1 - 711{,}591{,}369v_2 \leq 0$$
$$542{,}733{,}817u_1 - 132{,}752{,}229v_1 - 138{,}175{,}442v_2 \leq 0$$
$$1{,}299{,}518{,}856v_1 + 711{,}591{,}369v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Quebec 2019

$Max\ \theta = 362{,}006{,}150u_1$

Subject to

$$649{,}068{,}393u_1 - 230{,}175{,}833v_1 - 181{,}242{,}917v_2 \leq 0$$
$$1{,}884{,}974{,}133u_1 - 854{,}689{,}431v_1 - 608{,}010{,}480v_2 \leq 0$$
$$362{,}006{,}150u_1 - 89{,}891{,}228v_1 - 123{,}630{,}623v_2 \leq 0$$

$$89{,}891{,}228v_1 + 123{,}630{,}623v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Quebec 2020

$Max\ \theta = 414{,}794{,}857u_1$
Subject to

$$730{,}189{,}718u_1 - 248{,}689{,}687v_1 - 202{,}960{,}631v_2 \leq 0$$
$$2{,}030{,}124{,}141u_1 - 966{,}548{,}414v_1 - 608{,}487{,}946v_2 \leq 0$$
$$414{,}794{,}857u_1 - 103{,}156{,}388v_1 - 123{,}095{,}007v_2 \leq 0$$
$$103{,}156{,}388v_1 + 123{,}095{,}007v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Quebec 2021

$Max\ \theta = 478{,}589{,}953u_1$
Subject to

$$775{,}299{,}181u_1 - 252{,}363{,}903v_1 - 226{,}567{,}822v_2 \leq 0$$
$$2{,}214{,}253{,}122u_1 - 1{,}104{,}545{,}402v_1 - 603{,}136{,}222v_2 \leq 0$$
$$478{,}589{,}953u_1 - 137{,}123{,}650v_1 - 119{,}654{,}567v_2 \leq 0$$
$$137{,}123{,}650v_1 + 119{,}654{,}567v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Quebec 2022

$Max\ \theta = 520{,}026{,}432u_1$
Subject to

$$837{,}340{,}045u_1 - 264{,}232{,}035v_1 - 233{,}007{,}324v_2 \leq 0$$
$$2{,}517{,}240{,}579u_1 - 1{,}208{,}202{,}267v_1 - 649{,}617{,}004v_2 \leq 0$$
$$520{,}026{,}432u_1 - 141{,}540{,}012v_1 - 129{,}356{,}875v_2 \leq 0$$
$$141{,}540{,}012v_1 + 129{,}356{,}875v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

DEA model for Quebec 2023

$Max\ \theta = 542{,}733{,}817u_1$
Subject to

$$883{,}540{,}756u_1 - 262{,}680{,}674v_1 - 240{,}665{,}144v_2 \leq 0$$
$$2{,}715{,}343{,}184u_1 - 1{,}299{,}518{,}856v_1 - 711{,}591{,}369v_2 \leq 0$$
$$542{,}733{,}817u_1 - 132{,}752{,}229v_1 - 138{,}175{,}442v_2 \leq 0$$
$$132{,}752{,}229v_1 + 138{,}175{,}442v_2 = 1$$
$$u_1, v_1, v_2, \geq \varepsilon$$

The above mathematical equations were solved using Python's Pyomo Library (queries attached in the Appendix), and the results are reported in the following table.

Table 16- Efficiency score of each DMU for different scenarios

		DMU		
		BC	**Ontario**	**Quebec**
Scenario	***2019***	1	1	1
	2020	1	0.927356472	1
	2021	0.880219281	0.917862352	1
	2022	0.893915332	0.963898794	1
	2023	0.934667996	0.971489329	1
Average Efficiency		**0.941760522**	**0.956121389**	**1**

The table displays the efficiency scores for each DMU across five different years, from 2019 to 2023, as well as their average efficiency over this period. An efficiency score of 1 indicates that the DMU is operating on the efficiency frontier, meaning it is fully efficient relative to the other DMUs being evaluated. Scores below 1 indicate inefficiency, with the value representing the proportion of maximum possible efficiency achieved.

In 2019, all three DMUs, namely BC, Ontario, and Quebec, achieved an efficiency score of 1. This result indicates that during this year, all three provinces were operating at maximum efficiency. Moving into 2020, Quebec and BC remained fully efficient with scores of 1. However, Ontario's efficiency dropped slightly to 0.927356472. This change suggests that Ontario was operating at approximately 92.74% of its potential efficiency during this period. In

2021, Quebec maintained its efficiency score of 1, whereas BC and Ontario experienced declines in their efficiency. BC's efficiency score fell to 0.880219281, and Ontario's efficiency score dropped to 0.917862352. These scores indicate that BC and Ontario were less efficient in 2021 than Quebec. By 2022, Quebec sustained its full efficiency with a score of 1. BC and Ontario showed improvements, with efficiency scores rising to 0.893915332 and 0.963898794, respectively. This improvement suggests a positive trend in the efficiency of these provinces. Finally, in 2023, Quebec again achieved a score of 1, reflecting full efficiency. BC and Ontario continued to improve, with their scores increasing to 0.934667996 and 0.971489329, respectively. These results indicate a consistent upward trend in the efficiency of BC and Ontario.

Table 17- Ranking of DMU's according to the efficiency measure

DMU	Average Efficiency	Rank
Quebec	1	1
Ontario	0.956121389	2
BC	0.941760522	3

Table 17 presents the average efficiency scores and rankings for three provinces over the specified period. Quebec achieved the highest average efficiency score of 1, securing the 1st rank. Ontario followed with an average efficiency score of 0.956121389, placing it in the 2nd rank. British Columbia had the lowest average efficiency score of 0.941760522, resulting in the 3rd rank. This indicates that Quebec consistently performed at full efficiency, while Ontario and BC, though improving over time, still have some efficiency gaps to address.

Conclusions

This paper is aimed at applying DEA in supplier selection in the supply chain industry in the presence of multiple scenarios. DEA is a non-parametric approach for assessing the relative efficiency of DMUs based on input–output ratios relative to their peers.

The research uses data from greenhouse, sod, and nursery operations across Canada, sourced from Statistics Canada in 2024. The DEA model evaluates the efficiency of these operations over multiple years, specifically focusing on three key provinces: British Columbia, Ontario, and Quebec.

The output variable in the analysis is the total value of greenhouse products produced, while the input variables include various operating expenses incurred by specialized greenhouse producers, such as expenses related to specialized greenhouse vegetables and flowers.

The DEA results indicate that Quebec has consistently been the most efficient province from 2019 to 2023, achieving full efficiency each year. While Ontario and British Columbia have experienced periods of inefficiency, they have shown improvement over time. However, their average efficiency scores suggest that there is still room for enhancement to reach the level of efficiency consistently maintained by Quebec.

As this study introduces a new application of DEA with the incorporation of a multi-scenario approach in order to capture the variation and uncertainty within the greenhouse, sod, and nursery operations in Canada which will improve the quality and strength of efficiency analyses relevant to agricultural settings.

Limitations of the study

The research employs a static analysis approach by evaluating efficiency over multiple years but treating each year as a separate scenario. This method may not fully capture the dynamic nature of supplier performance and supply chain operations over time. A longitudinal analysis considering the evolution of efficiency scores could provide deeper and more meaningful insights.

Another limitation is that, with regard to the DMUs under analysis, homogeneity needs to be assumed. In this case, the provinces of British Columbia, Ontario, and Quebec are presumed to be homogeneous and comparable. However, regional policy differences, market conditions, or other contextual features may be such that their efficiency scores really are incomparable. If these contextual differences are not accounted for, then skewed results may appear.

Moreover, the research has only been targeted at greenhouse, sod, and nursery operations. While this gives an insight into this particular industry in great depth, generalization cannot be made for other industries or sectors that diverge in characteristics and dynamics.

The inherent assumptions of the DEA model also present limitations. DEA assumes the convexity of the production possibility set and that all DMUs operate under similar conditions. These assumptions may not always hold true, potentially affecting the robustness and reliability of the results. Moreover, DEA primarily deals with quantitative data, which means that qualitative factors such as supplier relationships, strategic importance, and innovation capabilities are not directly considered. These qualitative aspects can be crucial in making informed supplier selection decisions.

The geographical scope of the research is also limited to three provinces in Canada. Expanding the geographical scope to include more regions or countries could provide a broader understanding of efficiency in greenhouse operations. Finally, the temporal relevance of the data is a concern, as the data used is up to 2023. There may have been significant changes in the industry or external environment since then, and continuous updating of the data and re-evaluation of the models are necessary to maintain the relevance and accuracy of the findings.

Future Potential of the study

Looking ahead, the research on the application of Data Envelopment Analysis (DEA) in supplier selection when multiple scenarios exists, offers substantial potential for future development and application. Future studies will be able to provide much better and more timely insights into the efficiency of suppliers if they can leverage real-time data and enhance the dynamic capabilities of DEA. This broader geographical scope, along with the increased diversity of industries, will enhance the generalizability of findings and, hence, provide more complete insights into supplier performance across contexts. This would lead to a more holistic evaluation framework, one in which the qualitative factors of relationships with suppliers, strategic importance, and innovation capabilities are integrated with quantitative metrics.

Moreover, employing advanced methodologies to handle multiple scenarios and uncertainties in inputs and outputs, such as stochastic DEA models or hybrid approaches combining DEA with other decision-making techniques, can further refine the assessment of supplier performance. By continuously updating the data and incorporating external factors like economic conditions and regulatory changes, future research can maintain relevance and provide actionable insights, ultimately aiding organizations in making more informed and resilient supplier selection decisions.

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Appendix

```
# Step 1: Install Pyomo
!pip install pyomo

# Step 2: Import necessary libraries
from pyomo.environ import *

# Step 3: Define the model
model = ConcreteModel()

# Step 4: Define the variables
epsilon = 1e-6
model.u1 = Var(domain=NonNegativeReals, bounds=(epsilon, None))
model.v1 = Var(domain=NonNegativeReals, bounds=(epsilon, None))
model.v2 = Var(domain=NonNegativeReals, bounds=(epsilon, None))

# Step 5: Define the objective function
model.obj = Objective(expr=649068393 * model.u1, sense=maximize)

# Step 6: Define the constraints
model.con1 = Constraint(expr=649068393 * model.u1 - 230175833 * model.v1 - 181242917 * model.v2 <= 0)
model.con2 = Constraint(expr=1884974133 * model.u1 - 854689431 * model.v1 - 608010480 * model.v2 <= 0)
model.con3 = Constraint(expr=362006150 * model.u1 - 89891228 * model.v1 - 123630623 * model.v2 <= 0)
model.con4 = Constraint(expr=230175833 * model.v1 + 181242917 * model.v2 == 1)

# Step 7: Solve the model
!apt-get install -y -qq glpk-utils
solver = SolverFactory('glpk')
result = solver.solve(model, tee=True)

# Step 8: Print the results
model.display()
```