Sales Forecasting Using Classical and Machine Learning Approaches – A Comparative Study

University Canada West

MBAR 661 Consulting/Research Project

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Table of Content

List of Tables

List of Figures

Abstract

This research makes a comparison of sales predictions from classical statistical models with predictions from machine learning and deep learning models. Since sales forecasting is an activity that has an integral impact on an organization, the need to establish more accurate forecasting methods has prompted a large number of studies on the subject, in turn driving the creation of new models, however, one issue that was displaced is the error metrics of these predictions and their impact on the business sector.

Ten different forecasting models were used to obtain the predictions of four datasets, and their accuracy was measured using classical metrics such as ratio coefficients and vertical error metrics, and new theories such as peak similarity were also considered to establish the horizontal error of the predictions. The results show that classical statistical models showed the best metrics and coefficients, and that the choice of the best prediction model is not necessarily consistent with the best results for each metric. These results demonstrate the existence of under-studied error metrics with results that have an impact on the business world and may affect the choice of forecasting models.

Key Words: Sales forcasting, comparison, clasiccal statistics model, machine learning, deep learning, horizontal error, vertical error, peak similarity.

Introduction

Forecasting is a much-needed tool in everyday life in the current market; one can see its use in various industries like the agriculture industry (Idigova et al., 2023), retail (Rogermann et al., 2023), manufacturing (Kmiecik et al., 2022) whether it is for understanding sales, demand, or inventory assessment. Many operations/decisions are based on such predictions. Long-term forecasting will require more considerations such as long-term impacts on the business/company/area, how the market might change, whereas short-term forecasting might not be subjected to that many factors (Ping et al., 2023). Whereas short-term forecasting has its own implications and applications, such as in case of water supply demand in a city, sales spike for a new product launch or promotion (Guoxuan et al.,2023). Any business would want to estimate or guess its future sales so that it can prepare. Similarly, any company or organization would want to understand what their clients/users might need in future so that they can prepare well from now on. We have multiple concepts in that regard, like economic order quantity (EOQ), which can be used to improve profits, meet demand. (Tesalonika et al., 2023). This is also a forecast, and it is usually on a short-term basis.

Forecasts may affect not only the immediate department/company but also its effect can be felt for various other areas. For example, in order to meet the demand for a particular type of beverage during summer, it will be evident that the beverage company should increase its production. But in order to do that, the respective fruits company and, by extension, the plant seeds, fertilizers, and so on, should all be increasing their production (Technavio Research, 2020). This can happen if we expect/guess the demand of the future. When discussing sales forecasting, the demand will dictate how much product should be at the store/company to fulfill the needs of the clients. If the product on hand is in excess, then the business would have wasted time getting the product to the store and keeping the other products waiting, which could have met the needs of the clients better than the product that is in excess. On the other hand, if the product is short of stock, then the business loses the revenue that could have been generated from the excess demand. Also, other aspects, like inventory costs, labor costs, etc., could be controlled by a good forecast (Mascle et al., 2014).

When we know or predict the demand/sales for a particular company or product, decisions like how much product one needs to have it ready can be taken along with it, and other decisions like how that particular company can affect that future prediction can also be taken. For example, decisions like announcing a sales offer or promotion to improve sales, reducing the price of the product, changing the placement of the product, advertising, can be used to improve profits for the company (Rajaram et al., 2003). So, companies might be interested in understanding what their sales are looking like in future not only to scale themselves up/down but also to influence the market for their benefit.

Such knowledge of sales will have an impact on short and long-term decisions of a business, the importance of a high degree of certainty has been an important field of study in academic research, and the development of forecasting models has grown since statistical model to Machine Learning (ML) models, Deep Learning (DL) models, and hybrid (classic statistical with ML/DL) models (Makridakis & et al, 2020).

Since 1979, when studies made by Makridakis and Hibon found that predictions made by Brown's exponential smoothing adjusted by seasonality model were the most accurate over many others complex models, with the possibility of increasing this accuracy by averaging the predictions with other models (Makridakis et al, 2020), there is a question regarding the level of complexity willing to accept in a statistical model in order to increase its accuracy level; the use of advanced ML, DL or hybrid models represents a new challenge: the increasing complexity and cost at the moment of forecasting.

In order to illustrate the previously mentioned, we will use the results of the Makridakis Competition (M Competition) as an example; acknowledging the previously mentioned benefits of accurate forecasting in business, the M Competition proposes challenges in order to boost the studies and use of new trends in forecasting models for over 40 years (Makridakis et al, 2020). The M Competition, in its 4th edition (M4), made evident the increase in forecasting accuracy, which is represented by the decrease of the Symmetric Mean Absolute Percentage Error (sMAPE), due to the use of ML, DL and hybrid models; but also made evident the increasing complexity and cost of it, as it is seen in Figure 1; which, shows that increasing the training time of an ML, DL, or hybrid model reduces the sMAPE, thus increasing the accuracy of the model.

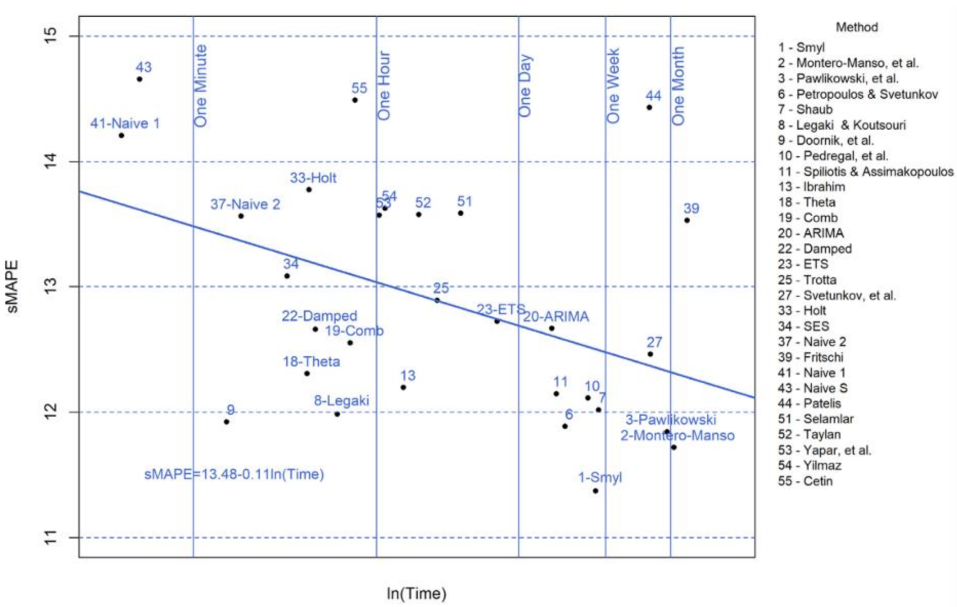

Figure 1. Negative Correlation Between the sMAPE and Training Time of ML/DL Models in M4 (Makridakis, Spiliotis, & Assimakopoulos, 2020)

Moreover, academic researchers in different fields have also made improvements in studies related to forecasting techniques, proposing new metrics for analyzing errors and understanding the effect of peak values in time series data. In this sense, the concept of Input Imitation (Zaji et al, 2019) was developed, indicating and demonstrating that peak values of a data series affect the forecasted values, causing a horizontal bias (Seen in Figure 2). Due to the presence of peak values in the time series that might be inducing bias in the forecasted values, this concept alerts to the existence of a bias, highlighting the importance of the horizontal analysis that should be measured and analyzed in parallel with the vertical analysis, in order to obtain metrics that indicate the real situation of the model and the forecasted values.

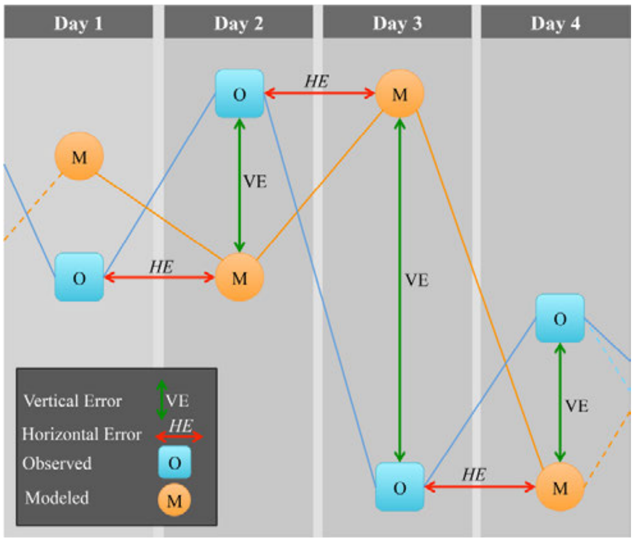

Figure 2. The input imitation problem in time series forecasting (Bonakdari et al., 2019).

Considering the aforementioned, this study aims to analyze the accuracy of predicted values using classical methods (Naïve, Moving Average, Exponential Smoothing, Autoregressive, ARIMA, and SARIMA), machine learning methods (Multilayer Perceptron, Decision Tree, and Support Vector Regression), and a deep learning method (Long Short-Term Memory). Using these methods, the relation coefficients (Coefficient of Determination, Correlation Coefficient), vertical error metrics (Mean Squared Error, Root Mean Squared Error, and Mean Absolute Error), and a horizontal error metric (Peak Similarity) of four different datasets will be obtained and will be performed a comparison of coefficients and metrics, analyzing comprehensively the consistency and accuracy of the prediction for each dataset.

Literature Review

From generic/naïve forecasting to forecasting based on data (Prescott,1922), sales forecasting changed over time. Prescott (1922) mentions how population growth was used as a base to forecast the sales of other products in the USA. As the data collection increased with the population growth and their differences in habits, the sales forecasting accuracy started dropping, the errors kept growing, and there raised a need for better forecasting techniques (Rothe, 1978). This results in better forecasting techniques like time series forecasting, regression analysis, classification and categorical forecasting and so on. More accurate techniques like time series forecasting help develop better decisions for managers and, by extension, the business (Mircetic et al., 2022).

As initial intuitive methods evolved into more complex forecasting; not only the forecasting method became efficient, but it also started considering the historical data in a systematic manner to become more accurate, and more dependent on quantitative methods. But, with the abundance/huge volume of the data that helps in forecasting that also covers the dependency on multiple variables, the accuracy also started dropping. And any manipulation of such forecasts generated by machines took work, which drove the users further away from the quantitative techniques in the early onset period of computers (Lawrence, 1983). Machine Learning accelerated the sophistication of forecasting as the world moved more toward it. Programs that can learn the intricate patterns of the data and thus forecast the probabilistic values for the data became more efficient with huge amounts of customer data based on different market segmentations (Chase, 2016). Although those could be somewhat biased, the professionals overseeing the development of machine learning and the predictive analysis techniques to forecast sales will help get the required results for the managers/businesses.

However, even with advanced techniques in forecasting, it is important to note that no technique is 100% correct, due to various factors like the time lag from the predicted and the actual data points to unforeseen reasons that increase the error in the predictions. The error is the difference between the observed and modelled values of the sales. Having different types of error calculation methods like Root Mean Square Deviation (RMSE), Mean Absolute Error (MAE), R^2 and as such will help models like AutoRegressive Integrated Moving Average (ARIMA), Seasonal AutoRegressive Integrated Moving Average (SARIMA), to become more accurate against these errors (Ramos et al., 2015). A vertical error, where the difference between actual and predicted value, has been discussed multiple times in research articles (Bannister, 2008; Neilson et al., 2022) to minimize the gap, but the horizontal error is rarely discussed.

Considering the importance of sales forecasting of products and services, countless works were carried out in order to establish the appropriate forecasting method for the different time series of each product and the appropriate error measures for each method; because the influence of the last aforementioned affect the preference of use of each method (Aras et al, 2017). Table 1 shows a brief summary of academics researches performed in order to analyze the best model for time series forecasting.

Table 1. *Researches of Forecasting Methods Comparison*

Reference / Year of Publication	Statistic Model	ML/DL Model	Error Metric	Best Model
(Ren et al., 2016)	Autoregressive Integrated Moving Average (ARIMA) Pure Panel Data (PPD) Grey Models	Extreme Learning Model (ELM)	Mean Squared Error (MSE) Symmetric Mean Absolute Percentage Error (sMAPE)	PPD
(Aras et al., 2017)	ARIMA Exponential Smoothing (ETS) Autoregressive Fractionally Integrated Moving Average (ARFIMA)	Artificial Neural Networks (ANN) Artificial Neural Network Fuzzy Interference (ANFIS)	sMAPE Theil-U Root Mean Squared Error (RMSE) Mean Absolute Error (MAE) Mean Absolute PercentageError (MAPE)	Combined Forecasting
(Elmasdotter & Nystromer, 2018)	ARIMA	Long Short Term Memory (LSTM)	RMSE MAE	LSTM
(Benboubker et al., 2019)	ARIMA ETS TBATS model	Neural Network Autoregression (NNA)	Mean Absolute Scaled Error (MASE)	NNA
(Liu et al., 2020)	Markov Chain Grey Model	ELM Support Vector Machines (SVM) Minimum Description Length Neural Network (MDL – NN)	sMAPE MASE Revised Mean Absolute Percentage Error (RMAPE)	MDL - NN

Reference / Year of Publication	Statistic Model	ML/DL Model	Error Metric	Best Model
(Smolak et al., 2020)	ARIMA	Extra-Trees (ET) Random Forest (RF) Support Vector Regression (SVR)	RMSE MAPE Nash-Sutcliffe Index of Efficiency (EI)	RF
(Haselbeck et al., 2022)	ETS Seasonal Autoregressive Integrated Moving Average (SARIMA) SARIMA with external factors (SARIMAX)	ANN LSTM Lasso Regression (LR) Ridge Regression (RR) Elastic Net Regression (ENR) Extreme Gradient Boosting (XGBoost) Bayesian Ridge Regression (BRR) Automatic Relevance Determination (ARD) Gaussian Process Regression (GPR)	RMSE sMAPE MAPE	XGBoost
(Ensafi & et al, 2022)	ARIMA SARIMA Autoregressive Moving Average (ARMA)	Facebook Prophet LSTM Convolutional Neural Network (CNN)	MSE RMSE MAPE	LSTM
(Iaousse & et al, 2023)	ARIMA	SVR LSTM K-Nearest Neighbor (KNN)	MSE MAE RMSE	LSTM KNN

Regarding the information in Table 1, it is evident that there is no rule over choosing a specific statistic model or ML/DL model to forecast that fits every situation (Aras et al, 2017). In 2016 and 2017, researchers obtained conclusions that show the classical statistics model with better accuracy than ML/DL models; henceforth, the improvements in this field increase the accuracy of ML/DL models, also increasing the public attention and use of these forecasting methods (Alroomi et al, 2022); this is visible in the conclusion of Best Model of each research from 2018 and above.

An important point, also visible in Table 1, is the preference for using vertical error metrics in order to obtain the accuracy of each method; just two of nine of the mentioned research used MASE as an error metric, although it is a measure of vertical error relative to the naive method, it can be considered a step towards the search for error metrics that offer new perspectives for the evaluation of the values predicted by any method.

The growing awareness that considering only one error measure does not guarantee a correct analysis and consequent selection of an adequate prediction model, because each error measure has strengths and weaknesses (Shcherbakov et al, 2013), resulted in the need to create new measures that can overcome these weaknesses; thus, these metrics grew in complexity up to the Scaled Pinball Loss Function for quantile forecast (Makridakis eta al., 2022) as an example; all these new analyses consider the vertical error as the axis of analysis.

However, all these approaches give little relevance to the analysis of the horizontal error, and considering that new research made evident the input imitation problem (Zaji et al, 2019), the use of horizontal error metrics will help to identify upper and lower peaks in the data, which affects the accuracy of the forecasting, with implication in the real world application.

Methodology

Overview

The paper considers the issue that is existing in the current retail world whether it is at a store level or at a country level. The actual issue being unable to perform more accurate short-term forecasting based on the data available. This is due to various factors like dependency on multiple variables like economic impacts, and trend impacts. Owing to all these it will become more complex to do effective predictions which can help the management to make decisions to sustain the company's growth. Accordingly, four datasets are chosen at various levels of the retail sector in different parts of the world and different areas of retail. Once chosen, the datasets were analysed for any underlying issues like missing data, inconsistencies, or reliability of the data. Depending on all these the data is cleaned, divided into training and test sets and made ready for further analysis.

To understand and compare the forecasting capabilities of the existing methods two types of forecasting methods were chosen namely statistical methods containing Naïve, Auto Regressive, Exponential, Moving Average, ARIMA, and SARIMA, similarly ML/DL methods are chosen namely MultiLayer Perceptron, Support Vector Regression, Decision Tree, Long Short-Term Memory methods. Based on the training and test sets each of these methods were trained and made sure all the parameters are matching for the forecasting purposes. As the results are tabulated and converted into graphs for respective methods, all the results are compared with other methods to understand the pattern of the predictions and the alignment of the actual results to our goal of the paper.

Having considered peak similarity one of the important error metrics, the conclusions are drawn based on the comparison of existing error metrics and peak similarity, on how our assumptions/ideas will help businesses to make more informed decisions.

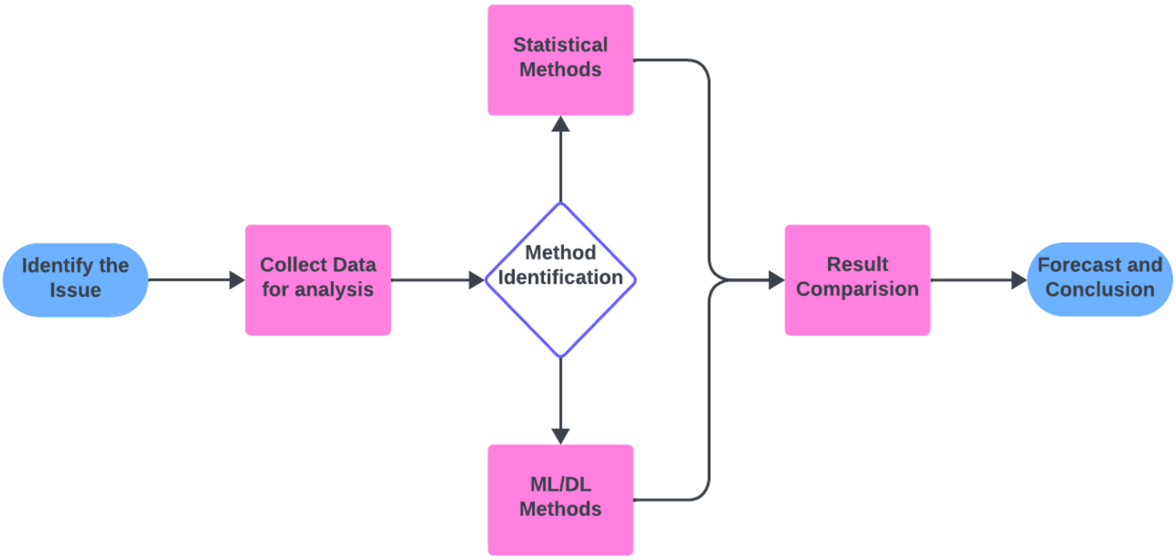

Figure 3. Flowchart of the research made by the author**.**

Limitations:

The present research has the following limitations:

1. As previously discussed, Horizontal Error is not valid in all time-series predictions. It depends on the case and needs to be chosen by the analyst accordingly.
2. Datasets used in the paper are adjusted by the provider for anonymity and business confidentiality purposes, so at times, we might be dealing with data points different from the original. This may lead to wrong predictions.
3. Datasets are limited; even though we have train and test sets, if the predictions need to be validated to date, it cannot be done.
4. The models and error metrics are not exhaustive; there might be other methods or error metrics that could give different results. Based on the scope and our discretion, particular methods and error metrics are used.
5. Real-time predictions could differ from the historical data; further research needs to be done in that area.

6. Horizontal error is not directly used to train the ML/DL models; rather, it is used to compare the results and the accuracy of predictions.

Datasets

In order to achieve the objectives of this research, the following datasets will be used:

Dataset 1 - Liquefied Gas sales in Bolivia

This research incorporates a dataset detailing monthly liquefied gas sales in Bolivia, sourced from the National Statistics Institute of Bolivia. Given the Bolivian government's long-term commitment to subsidizing petroleum derivatives in the domestic market (Ministerio de Hidrocarburos y Energias de Bolivia, 2023), accurately projecting sales volumes becomes crucial. This ensures a consistent domestic supply and helps anticipate the economic implications of the subsidy. Since the primary consumption of this fuel is by households, any shortage can have a significant social impact on the populace.

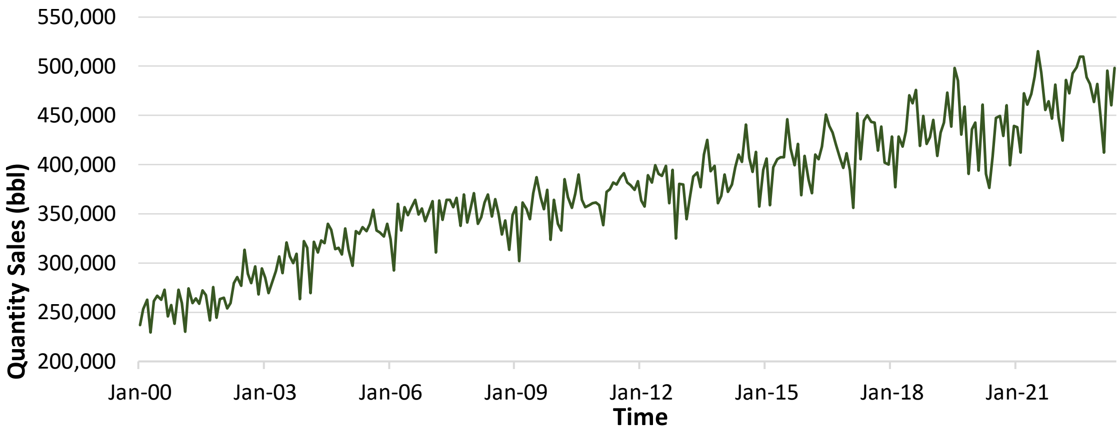

Figure 4. Plot of the Dataset 1

Dataset 2 – Diesel Oil sales in Bolivia

This research utilizes a dataset detailing monthly diesel oil sales in Bolivia, sourced from the National Statistics Institute of Bolivia. Notably, these sales are subsidized by the Bolivian government, with projections extending long-term (Ministerio de Hidrocarburos y Energias de Bolivia, 2023). Diesel fuel is predominantly consumed by heavy transport and industrial machinery. Consequently, its availability has a direct impact on sectors such as agriculture, industry, and transportation.

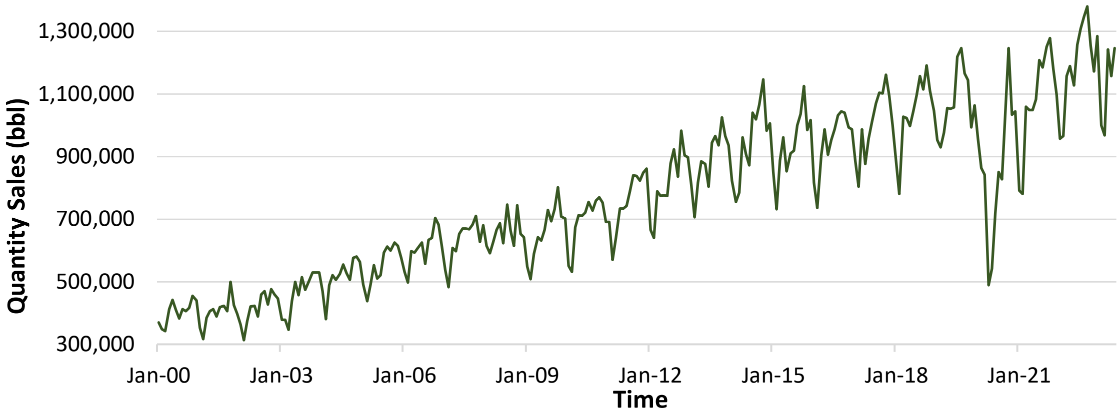

Figure 5. Plot of the Dataset 2

Dataset 3 – Brazilian Retailer

This dataset is extracted from the website Kaggle and was uploaded by a profile named TEVEC Systems (Tevec Systems, 2017). As per the website the data was obtained from a top Brazilian retailer and modifications to the data are done in order to anonymize the retailer. Out of the provided values from the site, the dateline and sales are used due to its relevance to the paper. The idea of the original author of this dataset was to provide basis for the implementation of Machine Learning (ML) models. The paper uses the dataset on similar lines. As discussed previously, short term forecasting helps the retailers in multiple ways like reducing wastage, solving inventory issues like space, ordering, replenishing, Etc. The same issue was considered even by the author of the dataset.

The dataset has dates starting from 01-January-2014 to 31-July-2016 with column name as Date. The second column consists of Sales data with column name as Sales or Data. The dataset is taken as is and no further assumptions to adjust the units of the data to be in hundreds or thousands of dollars are done. All the data appears as it is, and in direct dollar format. In order to relate to a real-world retailer, the amounts could be increased to thousands or millions depending on the size of the retailer which is anonymized initially. Total number of rows in the data are 937. The sales data ranges from 0 to 542. The data misses 6 dates at different places, no adjustments were to these points. The zeroes in the data could be assumed as either low sales or sales information are removed for confidentiality purposes.

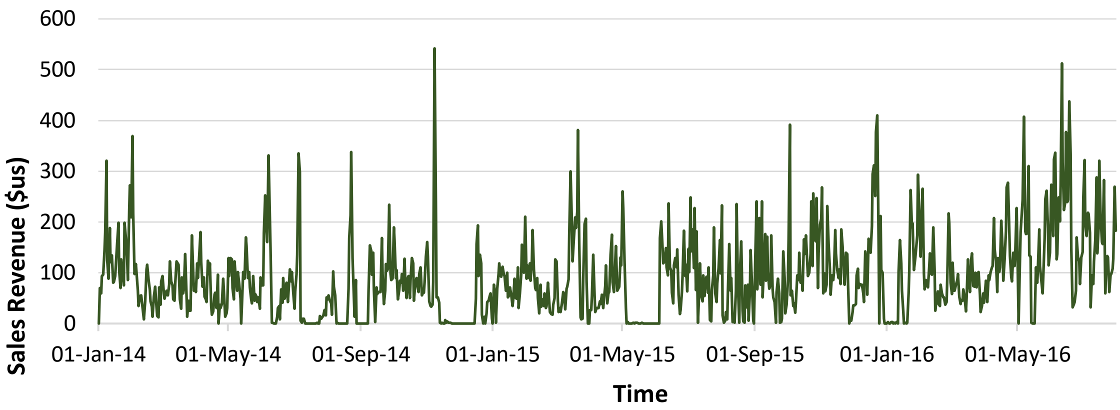

Figure 6. Plot of the Dataset 3

Dataset 4 – The United Kingdom Retail Sales

The dataset is extracted from the retail sales of the Great Britain having the country's data of all retail except automotive fuel. The dataset identifier code is J3L6. The website is of Britain's government, Office for National Statistics (ONS, n.d.). ONS is the UK's statistics producer, and the data found on the site is of open license. They release this information in periodic basis. This particular data is obtained from the dataset version dated, 18-Aug-2023 and the latest version is available from 22-Sep-2023.

It has two columns as Date and Data/Sales. The dates range from Jan-1994 to Jan-2016 with monthly intervals. There are total of 265 rows with sales starting from 16M to 34M. These are taken as they are, and currency conversion is not used as this is univariate data and is directly considered as dollars. Since the data is seasonally adjusted, the effect of other external factors can be considered low and the results from the predictions can be considered close to actual figures. The data keeps increasing from start to the end of the available dates with few variations. There are no missing or zero sales which helps the prediction methods produce better results. Since the data is for entire great Britain's but not for a single retail store or a retail company, this can not be seen in the context of direct inventory control but rather can be viewed as to aid in the allocation of resources or infrastructure at the country level by the government or private investors.

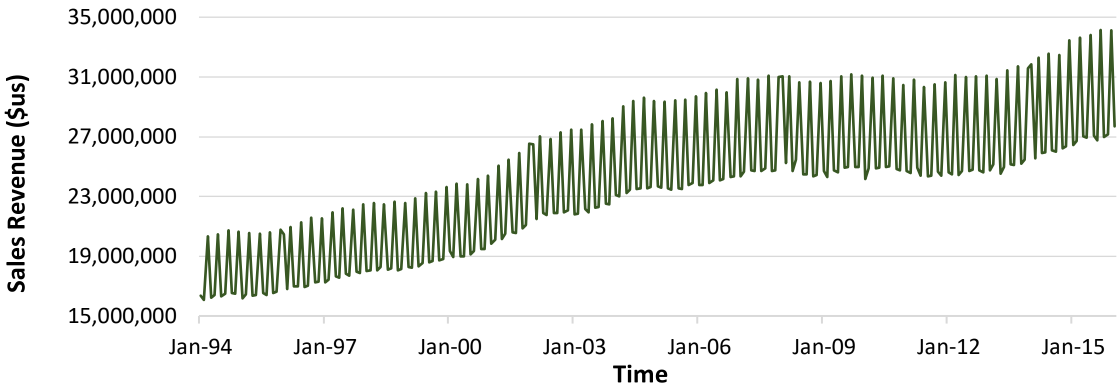

Figure 7. Plot of the Dataset 4

Descriptive Statistics

Considering the above, the descriptive statistics of the datasets used for this research are presented below.

Table 2. *Descriptive Statistics of Datasets*

Description	*Dataset 1*	*Dataset 2*	*Dataset 3*	*Dataset 4*
Mean	372,169	770,805	91	23,991,410
Median	370,712	745,235	76	24,335,865
Mode	407,825	690,176	0	
Standard Deviation	65,341	258,594	81	4,535,639
Range	285,862	1,066,278	542	18,067,974
Minimum	229,443	313,187	0	16,074,633
Maximum	515,305	1,379,465	542	34,142,607
Count	281	281	937	265

Forecasting Models

Naïve Method

The Naïve method may be considered one of the simplest forms of forecasting, which employs the method of using the immediate past actual output as the prediction for the current time period (Akpınar et al., 2017). For example, at time t, one needs to make a prediction of y_t and the actual output of the previous period is x_{t-1} then,

$$y_t = x_{t-1} \tag{1}$$

Moving Average

Moving Average can be seen as the average of the latest fixed number of data points for this paper (Hyndman, 2011). So, at any point in time, a new data point appears that is considered the latest and will be used to calculate the average starting from it and going backwards in the dataset. For example, the datasets are time series, so only the forward direction is considered for the scope of the project as progression. The latest data point is t, the previous one being t-1 and then t-3; a total of 3 data points are considered for taking the average. x_t,x_{t-1},x_{t-2} being the data points considered at a time, then the moving average at a specific time is,

$$MA_t = \frac{x_t+x_{t-1}+x_{t-2}}{3} \tag{2}$$

For this, the first three data points are ignored for making predictions due to the unavailability of the data. The rest are calculated under the column MA (Moving Average). The same calculations are used for both training and test data. MA6 and MA12 (a set of 6 and 12 values at a time, respectively) were calculated to show the correlation changes.

Exponential Smoothing

Alpha (α)/smoothing factor is being considered in this method for making the predictions. Making the predictions based largely on the recent data and less on further old data can be deemed as exponential smoothing (Billah et al., 2006). In this paper, α value is obtained by calculating the best possible value for a minimal error. Then, each prediction is based on its previous prediction as well as the actual value of the previous time period. For example, if the current time period is t the prediction is y_t, the previous prediction is y_{t-1}, and the previous actual value is x_{t-1} , then

$$y_t = \alpha x_{t-1} + y_{t-1}(1-\alpha) \tag{3}$$

Autoregressive Model

Autoregressive model forecasting by considering the historical data and their respective weights in predicting future values. The method employs different time lags; for example, Lag1 in this paper is considered as the immediate previous datapoint for the current datapoint (x_t). And three lags are considered for each prediction, and the lags are considered based on the correlation of their actual data points (Maatallah et al., 2015). The highest three correlated lags are considered for each dataset, and the test dataset uses the same lags as that of the train set. For calculating the weights, each lag coefficient is considered and used in the main equation as below,

$$y_t = \emptyset_a x_{t-a} + \emptyset_b x_{t-b} + \emptyset_c x_{t-c} \tag{4}$$

here, a, b, c represent the coefficients and lag values of Laga, Lagb, Lagc.

Autoregressive Integrated Moving Average (ARIMA)

This model combines both AR and MA to get the benefits of both the models of considering three different aspects like the order of AR (p) of the dataset which tells how many past lags were considered for the forecasting, degree (d) or the number of times the data needed to be differenced (subtract it with its past value) to make it stationary (which means that have constant mean and variance), and order of MA (q) which tells the number of past values used for the average (Ediger et al, 2007). An automated ARIMA package is employed for predicting p, d, q values by minimizing the AIC (Akaike Information Criterion), which tells the information lost when these particular values were used. Once the program finds the least AIC, those values are used in the ARIMA model to forecast the values of both training and test sets.

Seasonal Autoregressive Integrated Moving Average (SARIMA)

This method is the extension of the previous model. The concept of seasonality which tells the patterns in the given data over time. Considering the datasets used in the paper are retail sales, seasonality would be a better consideration for the predictions. Here, the seasonality 's' is considered for forecasting and similar packages and methods are used, like ARIMA forecasting. Once the p,d,q values are determined. Considering the seasonality s, P, D, Q (respective seasonal aspects) are predicted by minimizing AIC. And the forecasting is done for both train and test datasets.

For all Machine Learning models, lags were prepared as a separate function. Similarly, graphs and Peak Similarity (PS) functions were prepared in order to be used under each model to be called whenever required. Training and test data sets are divided accordingly, with 70% and 30% of data in each set, respectively. Employing different ML (Machine learning) models using Python packages by selecting appropriate parameters resulted in optimum results as per the scope of the paper.

Multilayer Perceptron (MLP)

MLP works based on the weighted connections, neurons, activation functions, and connection of all these neurons to form different layers from input to output, giving the scope for the network to learn the data and predict future sales accordingly. Different combinations of all these have been employed to get the optimal output (Armano et al., 2023), such as a total of 1 layer with 15 neurons for datasets 1 and 2 and 2 layers with 10 neurons each. The number of epochs/iterations are defined as 1000. Activation function ReLu (rectified linear activation unit) solver lbfgs, and ADAM (adaptive moment estimation) with a random state of 42 are used. Once all are defined by importing the MLP python package, the same is used for training using the train set and predicting the test set.

Decision Tree (DT)

DT develops the predictions based on the leaf nodes that will be split from the main node for making a decision based on the significance of the decision. For example, if a decision to select between two states needs to be made, then two leaves will be developed from a sample, giving each decision its respective weight and so on. Thus forming a tree-like structure (Chen et al., 2017). In this paper, a similar approach is used for the prediction by defining each parameter as follows. The maximum depth the tree can reach will be of 5 levels. Each node can be split into 3 child nodes. A leaf can have one sample. All the features in the data need to be considered. As done for MLP, a random initial state of 42 is selected.

Support Vector Regression (SVR)

The SVR model uses the input data to transform into higher dimensions in order to identify the patterns. It can be done using kernel functions, and this paper uses the polynomial kernel, which raises the power of the input data to introduce non-linearity (Chen et al., 2017). Then, a regularization of C=3 for datasets 1 and 2, and C=1 for dataset 3 and 4, indicating moderate regularization strength. Epsilon of 0.1 is used, which identifies the width of the tube, i.e., the error margin for the predictions. Combining all the imported SVR function through the package is used to predict the training and test sets accordingly. A general SVR function can be represented as below,

$$f(x) = \sum i = 1N(\alpha i - \alpha i *) \cdot K(x, xi) + b \tag{5}$$

here, f(x) is the output for a given input x. N is the number of support vectors. $\alpha i, \alpha i *$ are Lagrange multipliers. K is the kernel function; b is bias term.

Long Short-Term Memory (LSTM)

This method draws both the advantages of long-term as well as short-term memory by choosing the parameters appropriately and also avoiding the chances of overfitting while considering the historical data to train the model. It has different gates (forget, input and output) which determine the retention and passing of the information (Abbasimehr et al., 2020). In order to use the LSTM model, the data is reshaped into a 3 dimensional with a third dimension of size 1. Similar to the MLP, ReLu activation and ADAM solver are used. 100 units/neurons are defined for this function. And for the regression task, Mean Squared Error is considered. As the information is flown through each neuron, the memory is retained based on the parameters and adjusted accordingly as the new information appears, eventually producing the final predictions.

Relation Coefficients and Error Metrics

In order to identify the relevance of the use of classical or machine learning forecasting methods, the following coefficients will be used. In order to define each coefficient, we will refer to Error (E) as the difference between the actual value (y) and the predicted value (ŷ) of a dataset, which the following formula can define:

$$\mathrm{E} = \mathrm{y} - \hat{\mathrm{y}} \tag{6}$$

also, the mean of the actual values ($\bar{y}$) is the total sum of all the actual values, divided by the number of values (m), the following formula can define it:

$$\bar{y} = \frac{1}{m} \sum_1^m y \tag{7}$$

Coefficient of Determination (R^2)

The coefficient of determination indicates the proportion of the variance in the dependable variable explained by the independent variable. It is a value between 0 and 1; an R^2 of 1 means that the independent variable explains all the variance of the dependent variable. It can be obtained using the following formula:

$$R^2 = 1 - \frac{\sum_1^m (y - \hat{y})^2}{\sum_1^m (y - \bar{y})^2} \tag{8}$$

Correlation Coefficient (r)

The correlation coefficient indicates the linear relationship between two variables. It is a value between -1 and 1; a correlation coefficient of 1 indicates a perfect positive correlation, -1 indicates a perfect opposite correlation, and 0 indicates the inexistence of correlation between the values. As mentioned, to obtain the correlation coefficient it is necessary to have two sets of data (a, b) of the same amount (m); each set of data will represent a point within the linear relationship; it is also necessary to have the mean of the actual values on both datasets (a_{hat}, b_{hat}) can be defined by the following formula:

$$r = \frac{\sum_1^m (a - a_{hat})(b - b_{hat})}{\sqrt{\sum_1^m (a - a_{hat})^2 \sum_1^m (b - b_{hat})^2}} \tag{9}$$

In order to analyze the classical and machine learning approaches of forecasting methods, the following metrics will be analyzed.

Mean Squared Error (MSE)

Mathematically, the MSE is the sum of all the squared E of a dataset divided by the number of values. The following formula can also define it:

$$\mathrm{MSE} = \frac{1}{m} \sum_1^m (E)^2 \tag{10}$$

Root Mean Square Error (RMSE)

The RMSE is the root squared of the MSE; the following formula represents it:

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{m}} \sum_{1}^{\mathrm{m}} (\mathrm{E})^2} \tag{11}$$

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}} \tag{12}$$

Mean Absolute Error (MAE)

The MAE can be defined as the summation of all the absolute values of the error divided by the number of dates used in the dataset (m). it is represented by the following formula:

$$\mathrm{MAE} = \frac{1}{m} \sqrt{\sum_{1}^{m} |E|} \tag{13}$$

The metrics mentioned previously are part of the vertical metrics, which consider the error of the predicted values with the actual values in each period of time. Zero is the best possible value to this metrics. The different formulas to obtain them also creates the problem of inconsistency, this means that lower MSE or RMSE, not necessarily implies the lower MAE. Thus, a comprehensive analysis of all metrics are needed (Chicco & Warrens & Jurman, 2021). To offer a new point of view about the relevance of forecasted values of a dataset, the following metric will be used.

Peak Similarity (PS)

For the actual values (y), a peak will be defined as a higher or lower value considering a later value and a previous value. And for the predicted values (ŷ), a peak will be defined as higher or lower value considering a later value and a previous value; and a peak must exist in the actual values in the same frequency or period and must be in the same direction (higher or lower).

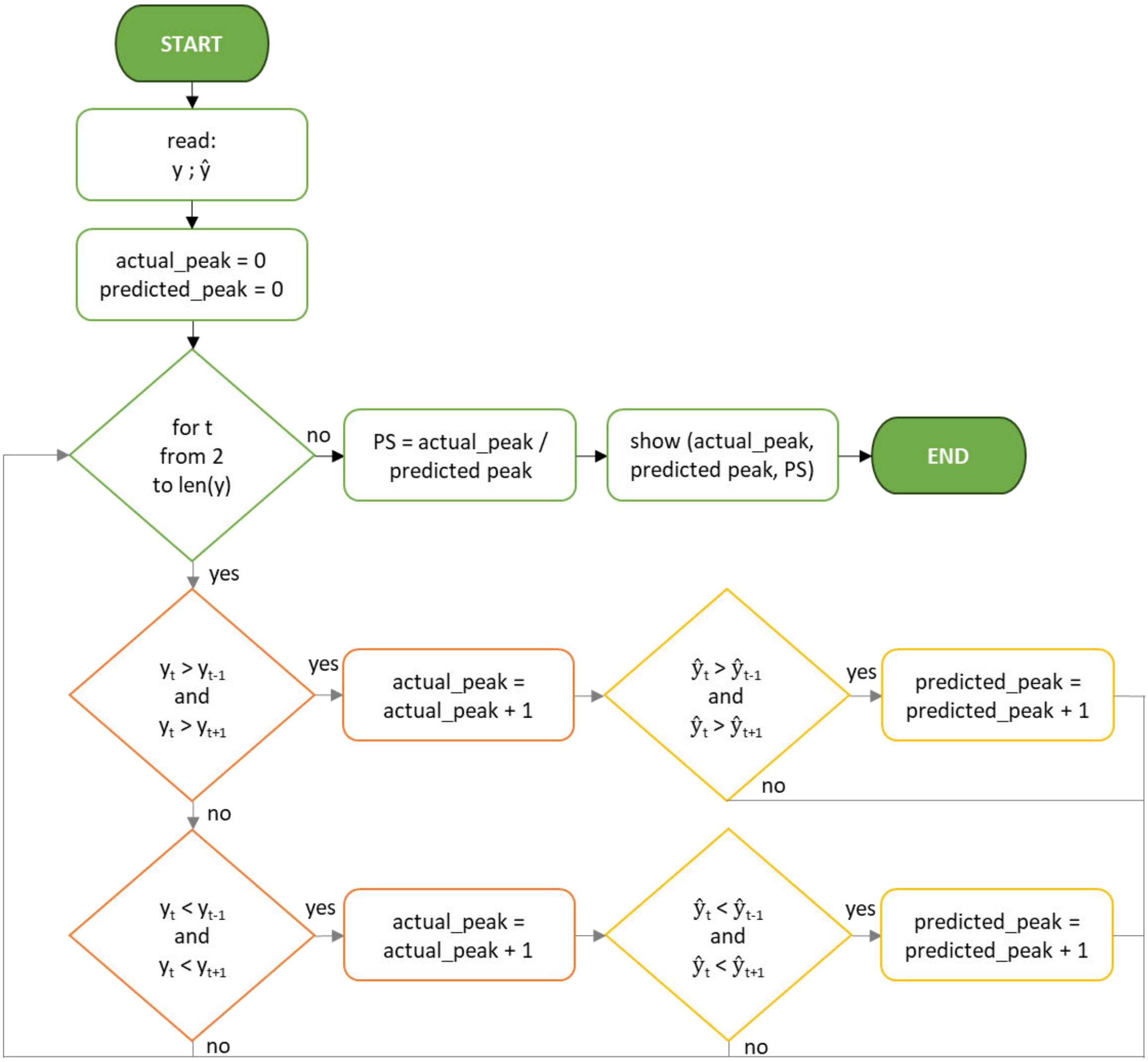

Figure 8. Peak Similarity Flow Chart

Rationale for Horizontal Error

As seen in the literature review, there are multiple papers that are based on various types of error metrics and very few that are actually based on Horizontal Error. Considering a retail store as an example, they might want to understand when they are going to see peaks or troughs in their demand so that they can prepare well for it. Especially during special occasions like holidays/promotional days, even if they miss the assumption by one day, the prediction can be considered as wrong due to the missed sales target. Or, in an occasion where there are signs of a flood, the government might want to know the precise time so that they can estimate the resources needed to evacuate the people and work accordingly. Based on this idea, Horizontal Error is important in such cases. It might not be the case for all types of time-series predictions; in cases such as the overall average error, it needs to be given higher importance than catering to the surges in demand. Also, in cases where there will be a lesser chance of sudden or unpredicted surges, Horizontal Error might not be an optimum choice (this case will be further discussed with data in the next sections of the report).

Results and Discussion

Once the prediction of the four datasets (shown in Appendix A, B, C, and D) were obtained, the relation coefficients and error metrics were also obtained (Appendix E); in order to perform the following analysis, the coefficients and metrics obtained from Dataset 1 will be used, considering that the mentioned coefficient and metrics cover the scope and goals of this research; important information obtained from other datasets will be mentioned in the discussion section.

Relation Coefficients Analysis

The predicted values' accuracy of Dataset 1, according to the coefficient of determination and correlation coefficient, are shown in the following figure (values closer to +1 mean better accuracy).

Train Accuracy	Model	Test Accuracy
0.784	SARIMA	0.604
0.886		0.777
0.860	Long-Short Term Memory	0.400
0.936		0.636
0.887	Autoregressive	0.398
0.942		0.631
0.868	Multilayer Perceptron	0.385
0.941		0.629
0.850	Moving Average	0.345
0.922		0.588
0.853	Exponential Smoothing	0.277
0.923		0.527
0.762	Naïve	0.238
0.873		0.488
0.873	Support Vector Regression	-0.197
0.935		0.649
0.759	ARIMA	0.000
0.871		-0.008
0.944	Decision Tree	-1.537
0.972		-0.075

Coefficient of Determination | Correlation Coefficient

Figure 9. Accuracy of Forecasting models considering relation coefficients – Dataset 1

As mentioned in Figure 9 for Dataset 1, it is observed that the high relation coefficients shown in the train set are not visible in the test set, which can be a sign of overfitting of the model or extreme values affecting the relationship between predicted and actual values, especially considering the Decision Tree and SVR models. SARIMA shows the higher relation coefficient's accuracy considering the test set, and the Decision Tree and ARIMA models show lower accuracy with the same parameters.

Vertical Error Analysis

Considering the vertical error metrics, the following figure presents the RMSE, MSE, and MAE obtained for Dataset 1 (Values closer to 0 mean lower error).

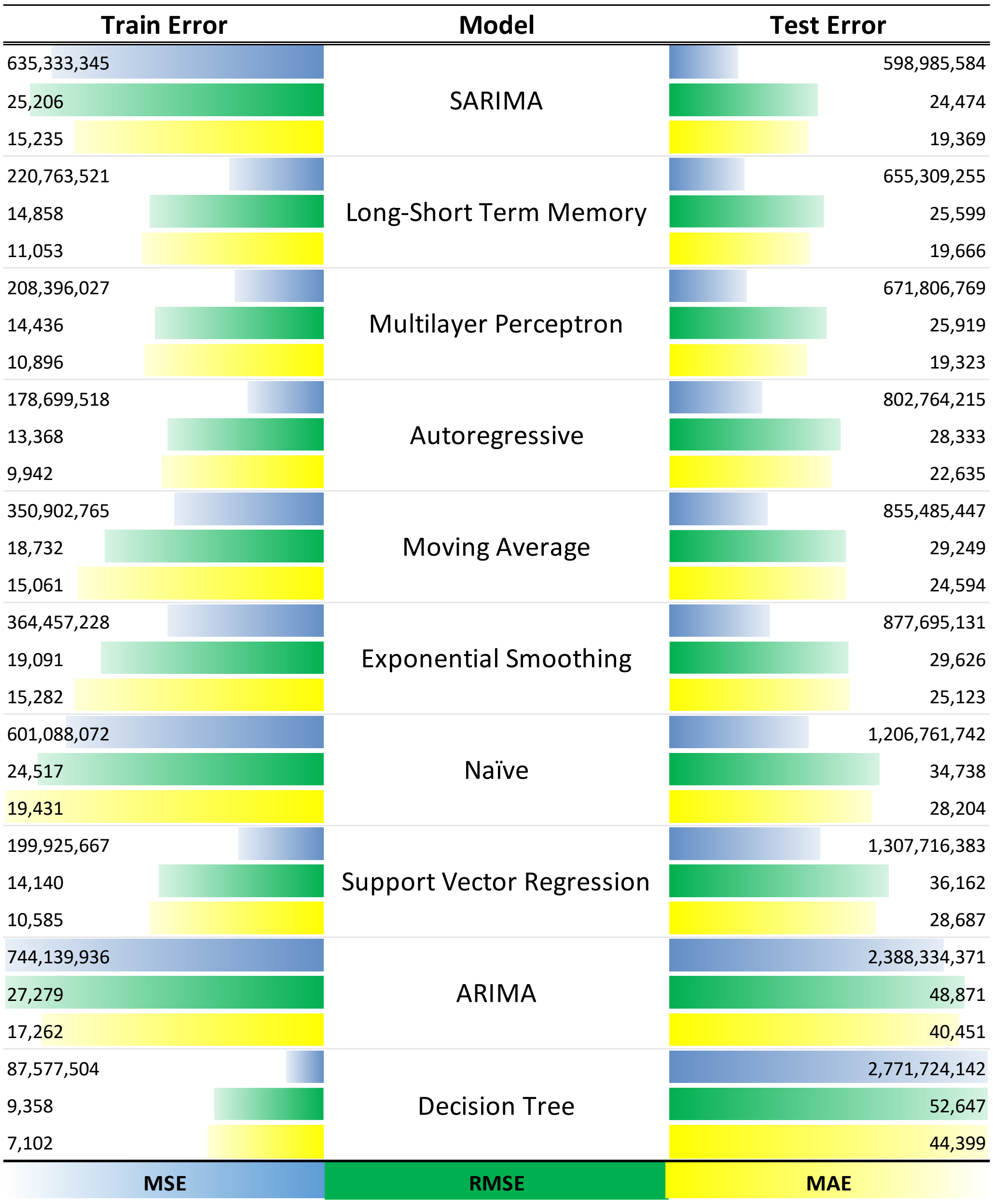

Figure 10. Accuracy of Forecasting models considering vertical error – Dataset 1

In Figure 10, it is visible that SARIMA shows the lowest error for the test data of Dataset 1, closely followed by LSTM and MLP models, same as relation coefficients, in the vertical error analysis the Decision Tree, and ARIMA model shows the higher error. Considering the relation coefficients and vertical error metrics' analysis, it is observable that both analyses are highly related and both metrics are used as accuracy metrics of predicted values, excluding metrics that can enhance the analysis of predicted values' accuracy. Also, it is observable that classical statistical forecasting models, are able to offer better accuracy than machine learning or deep learning models.

Horizontal Error Analysis

In order to perform the horizontal error analysis, the following information regarding peak similarity was obtained from Dataset 1 (values closer to +1 mean better accuracy).

Train Accuracy	Model	Test Accuracy
0.735	Autoregressive	0.667
0.701	Multilayer Perceptron	0.667
0.675	Support Vector Regression	0.667
0.624	Long-Short Term Memory	0.641
0.331	SARIMA	0.585
0.169	ARIMA	0.255
0.641	Decision Tree	0.179
0.127	Moving Average	0.111
0.000	Exponential Smoothing	0.000
0.000	Naïve	0.000

Peak Similarity

Figure 11. Accuracy of Forecasting models considering horizontal error – Dataset 1

As mentioned in Figure 11, considering the peak similarity as a horizontal error metric, it is obtained that AR, MLP and SVR share the higher accuracy, and Exponential Smoothing and Naïve Method share the lower accuracy under this parameter. The results of the horizontal error analysis show a different result than relation coefficients and vertical error metrics and must be analyzed according to the forecasting need, this analysis can conclude in using a forecasting method with higher accuracy in relation metrics and vertical error metrics, higher accuracy in horizontal error metrics, or balanced accuracy. Also, it is important highlight that for Dataset 1, a classical statistical model offers the best accuracy regarding horizontal error.

It is important to mention that errors like MSE, RMSE, and R^2 can be used to analyze the Vertical error, but Horizontal Error is generally hard to achieve due to its nature, like the dependency of more than one variable for the forecasting. For example, sales may change due to the holidays, new products, marketing, consumer habits, discounts, inflation, Etc. these factors are not easily accountable due to the simple nature of the forecasting models chosen. Thus, accounting for Horizontal Error would also be tricky. ML/DL models depend on the previous input for the prediction, which can make it difficult for the model to assume the correct peaks. In our four datasets, similar cases were shown with both statistical and ML/DL models; both achieved very few peak similarities, with the highest being 76%+ in Dataset 4. The concept of peak similarity itself can be further optimized with respect to dependency on other variables, standing in coherence with other error metrics, etc., along with other ML/DL forecasting issues.

In our datasets, both statistical as well as ML/DL models failed to achieve the balance. Hence, there is a need to develop methods which take into consideration all these metrics to build better predictions based on the business requirements.

Conclusion

Although it intuitively appears impossible to achieve even near-perfect Horizontal Error, a short-term forecasting consideration can make this idea more plausible. Unlike stock market changes or disaster situation forecasting, short-term retail sales forecasting could see less drastic changes from day to day or year to year. This gives the scope for improvement of peak similarity, which could benefit retail stores or even the retail sector of a country in various areas. Inventory can be effectively managed, and when it comes to smaller stores, it is not easy to manage or store high volumes of inventory in anticipation of sales peaks. Predicting the peaks with appropriate time gaps could allow the stores to plan accordingly and order the inventory at the right times, thus reducing storage costs (Rockeman, 2022). Labour underutilization or insufficiency can be avoided by predicting the peaks. Other factors can be brought into the picture to improve the troughs or take appropriate actions at the right time. In the case of bigger economic areas like the retail sector of a country, the infrastructure of the country can be arranged to facilitate the sales changes, i.e., imports/exports. On the other side, predicting peaks at the wrong time can reverse these effects and can lead to more wastage or shortage of resources than other non-peak times. The combined effect of this shortage / wastage can lead to a bigger impact on the profits of the companies.

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APPENDIX A

Plot and Correlation of Dataset 1 - Train Values and Predictions

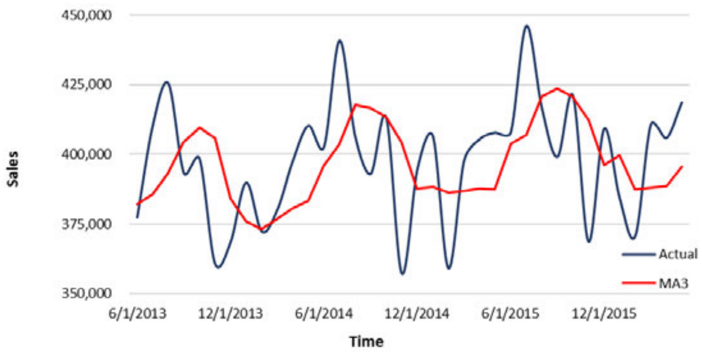


Figure A.1 Dataset 1 and Moving Average Predictions (36 last values)

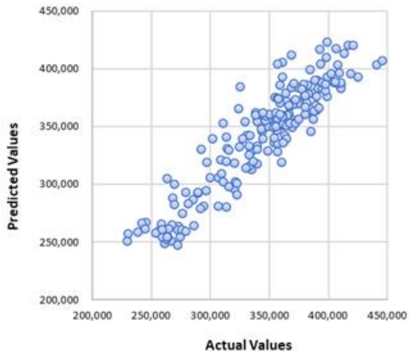


Figure A.2 Correlation between Dataset 1 and Moving Average Predicted Values

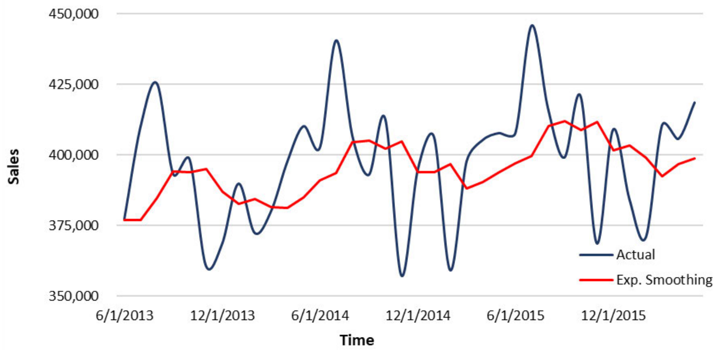


Figure A.3 Dataset 1 and Exponential Smoothing Predictions (36 last values)

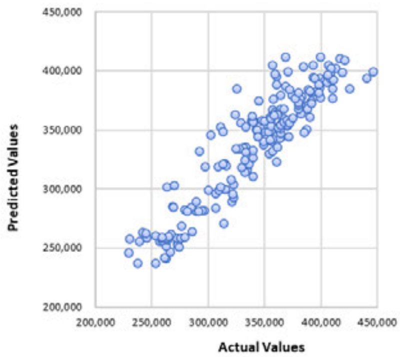


Figure A.4 Correlation between Dataset 1 and Exponential Smoothing Predicted Values

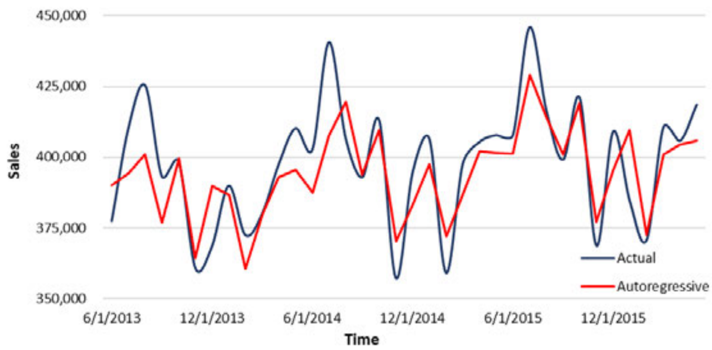


Figure A.5 Dataset 1 and Autoregressive Predictions (36 last values)

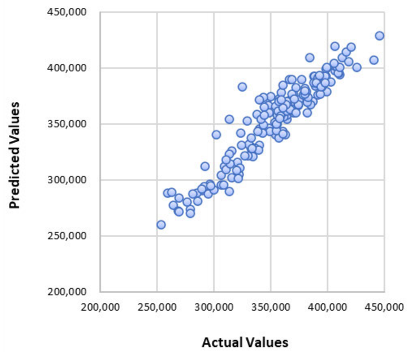


Figure A.6 Correlation between Dataset 1 and Autoregression Predicted Values

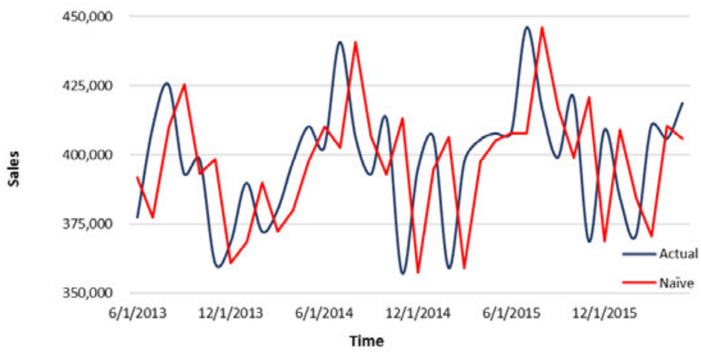


Figure A.7 Dataset 1 and Naive Predictions (36 last values)

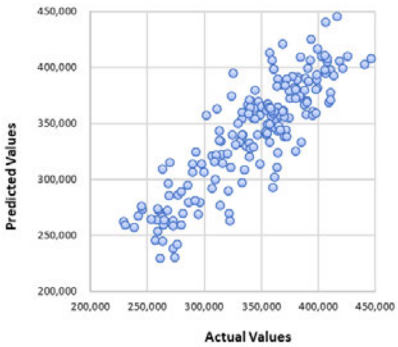


Figure A.8 Correlation between Dataset 1 and Naive Predicted Values

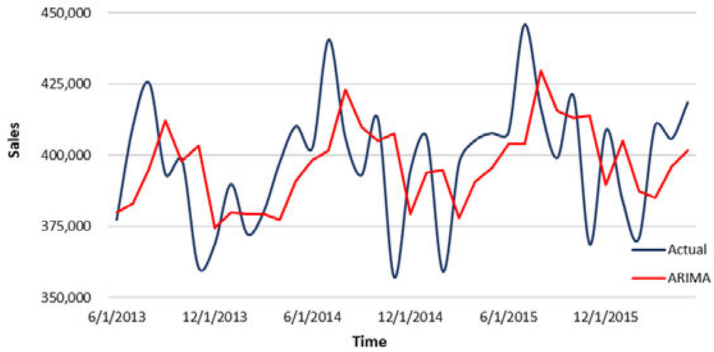


Figure A.9 Dataset 1 and ARIMA Predictions (36 last values)

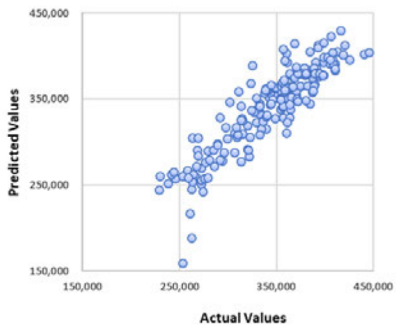


Figure A.10 Correlation between Dataset 1 and ARIMA Predicted Values

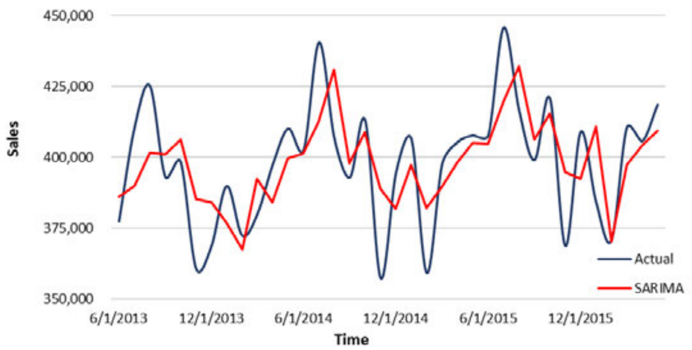


Figure A.11 Dataset 1 and SARIMA Predictions (36 last values)

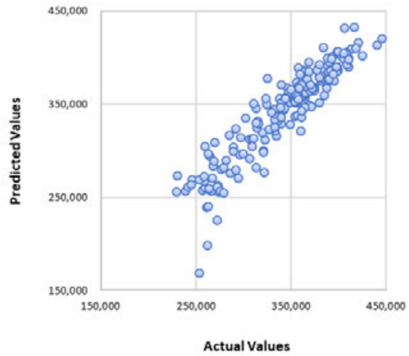


Figure A.12 Correlation between Dataset 1 and SARIMA Predicted Values

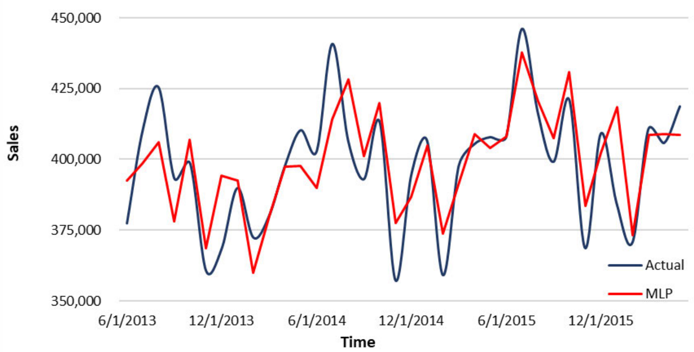


Figure A.13 Dataset 1 and MLP Predictions (36 last values)

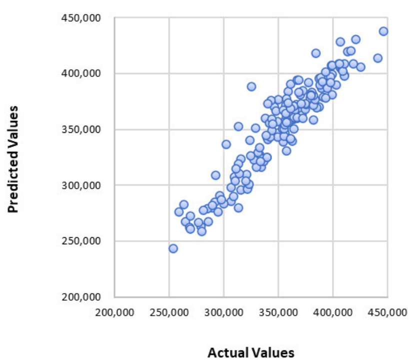


Figure A.14 Correlation between Dataset 1 and MLP Predicted Values

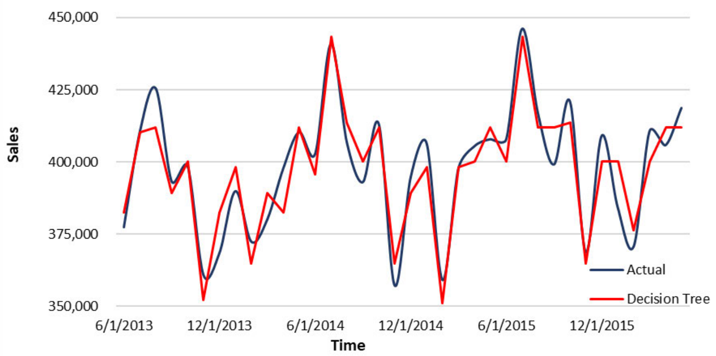


Figure A.15 Dataset 1 and Decision Tree Predictions (36 last values)

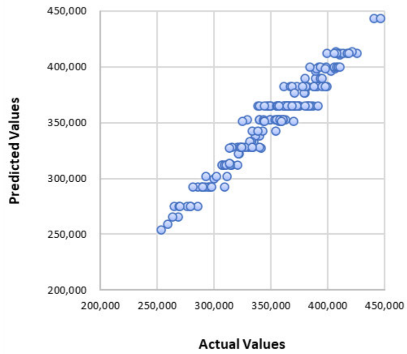


Figure A.16 Correlation between Dataset 1 and Decision Tree Predicted Values

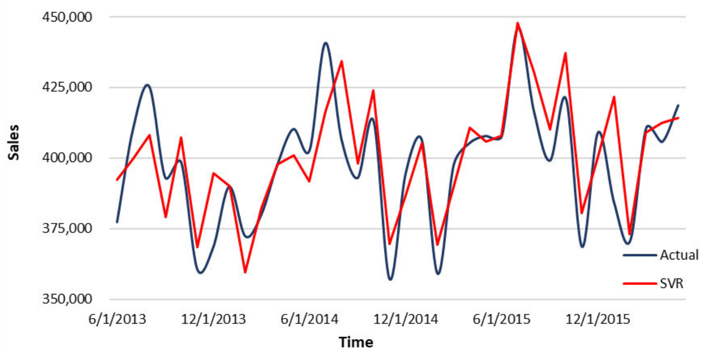


Figure A.17 Dataset 1 and SVR Predictions (36 last values)

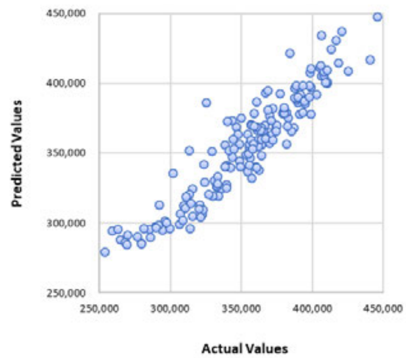


Figure A.18 Correlation between Dataset 1 and SVR Predicted Values

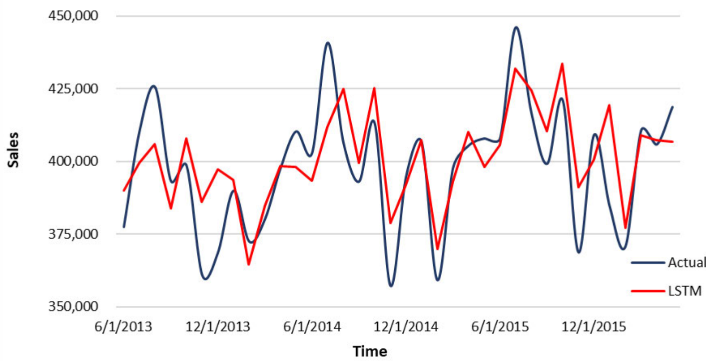


Figure A.19 Dataset 1 and LSTM Predictions (36 last values)

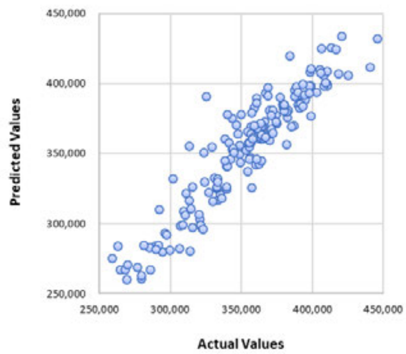


Figure A.20 Correlation between Dataset 1 and LSTM Predicted Values

Plot and Correlation of Dataset 1 - Test Values and Predictions

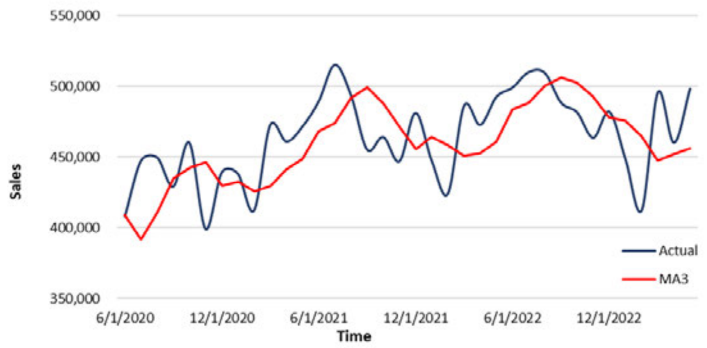


Figure A.21 Dataset 1 and Moving Average Predictions (36 last values)

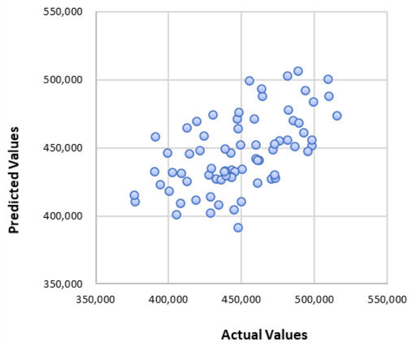


Figure A.22 Correlation between Dataset 1 and Moving Average Predicted Values

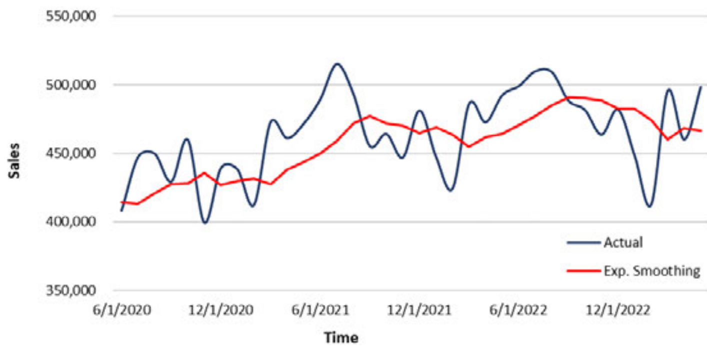


Figure A.23 Dataset 1 and Exponential Smoothing Predictions (36 last values)

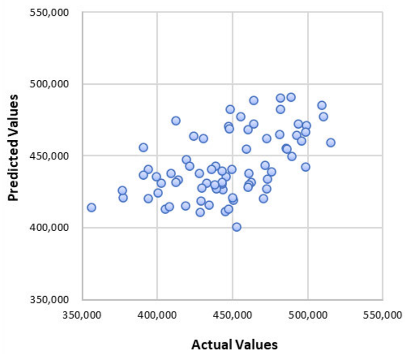


Figure A.24 Correlation between Dataset 1 and Exponential Smoothing Predicted Values

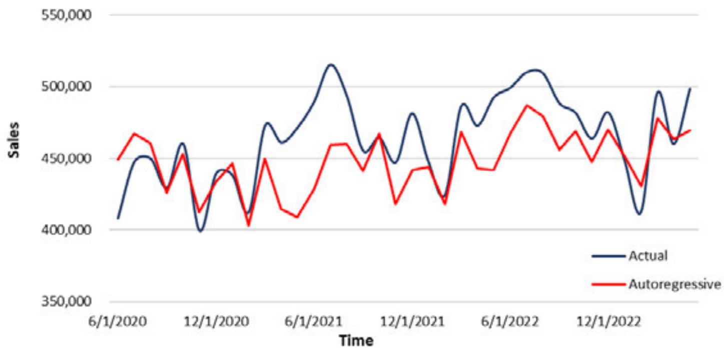


Figure A.25 Dataset 1 and Autoregressive Predictions (36 last values)

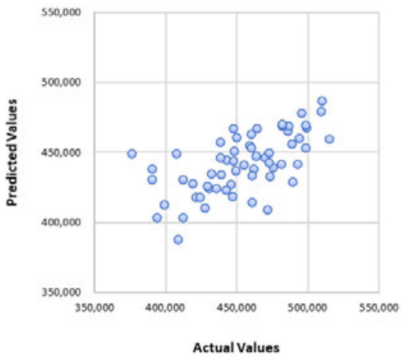


Figure A.26 Correlation between Dataset 1 and Autoregressive Predicted Values

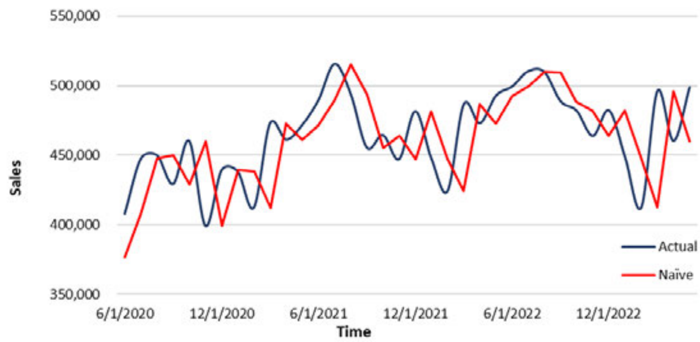


Figure A.27 Dataset 1 and Naive Predictions (36 last values)

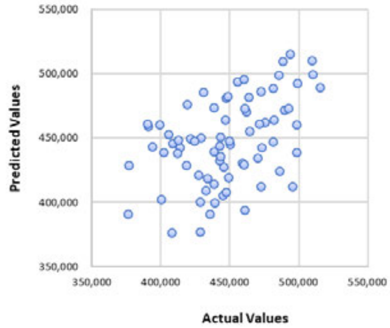


Figure A.28 Correlation between Dataset 1 and Naive Predicted Values

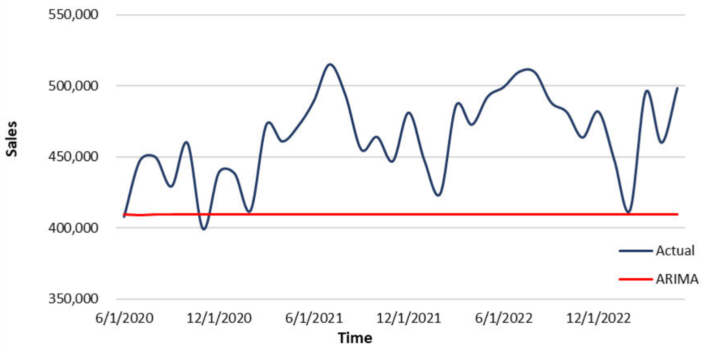


Figure A.29 Dataset 1 and ARIMA Predictions (36 last values)

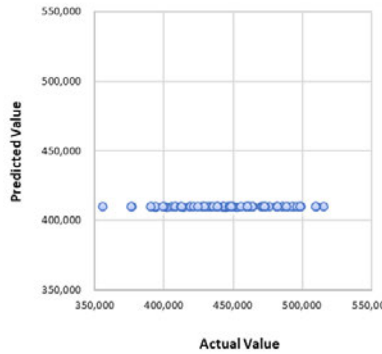


Figure A.30 Correlation between Dataset 1 and ARIMA Predicted Values

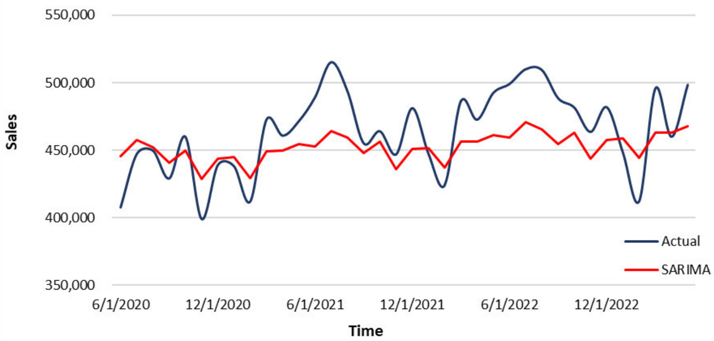


Figure A.31 Dataset 1 and SARIMA Predictions (36 last values)

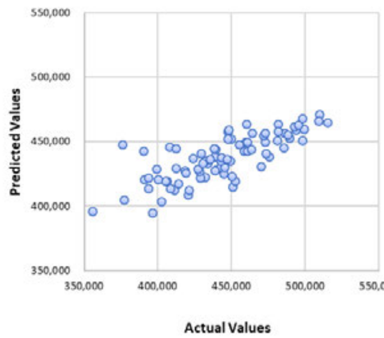


Figure A.32 Correlation between Dataset 1 and SARIMA Predicted Values

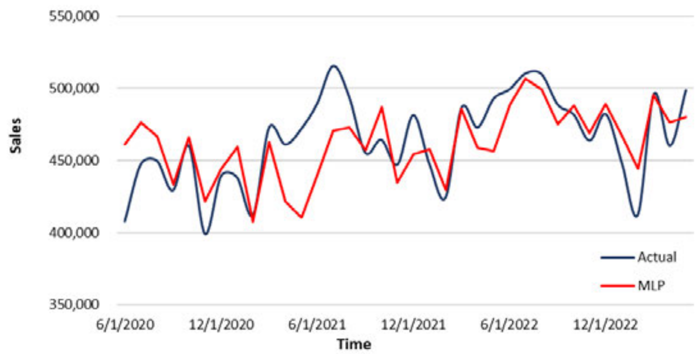


Figure A.33 Dataset 1 and MLP Predictions (36 last values)

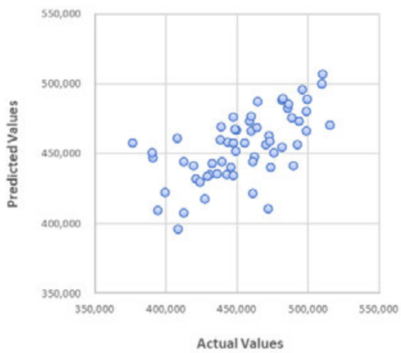


Figure A.34 Correlation between Dataset 1 and MLP Predicted Values

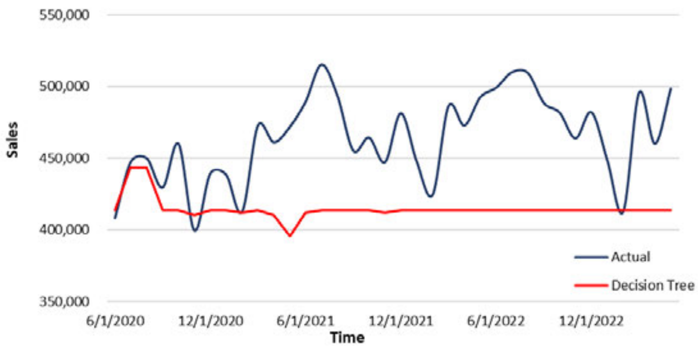


Figure A.35 Dataset 1 and Decision Tree Predictions (36 last values)

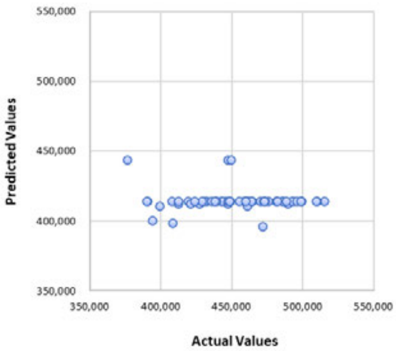


Figure A.36 Correlation between Dataset 1 and Decision Tree Predicted Values

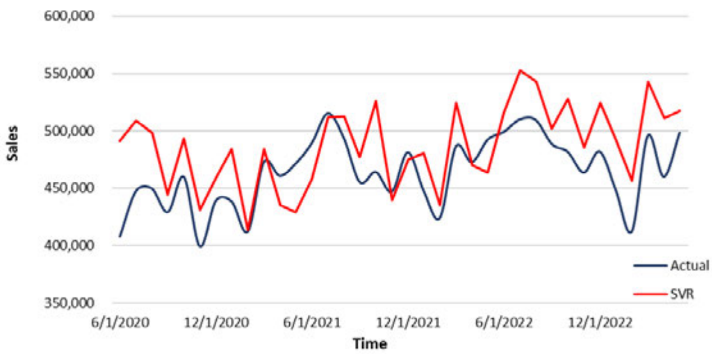


Figure A.37 Dataset 1 and SVR Predictions (36 last values)

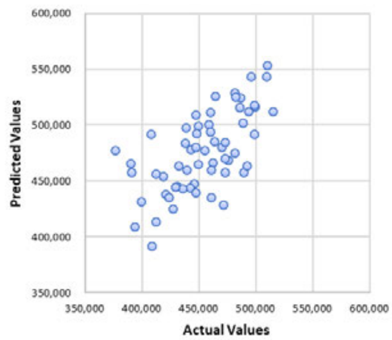


Figure A.38 Correlation between Dataset 1 and SVR Predicted Values

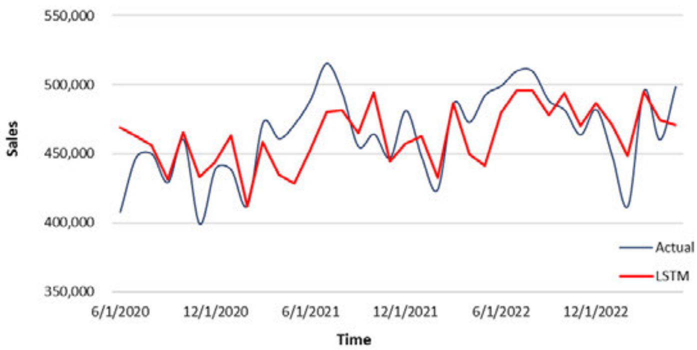


Figure A.39 Dataset 1 and LSTM Predictions (36 last values)

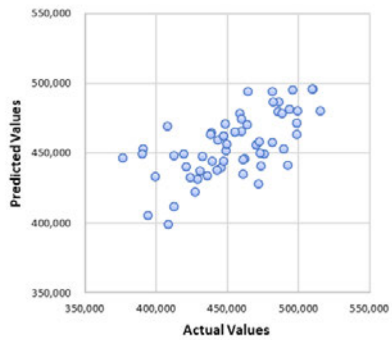


Figure A.40 Correlation between Dataset 1 and LSTM Predicted Values

APPENDIX B

Plot and Correlation of Dataset 2 - Train Values and Predictions

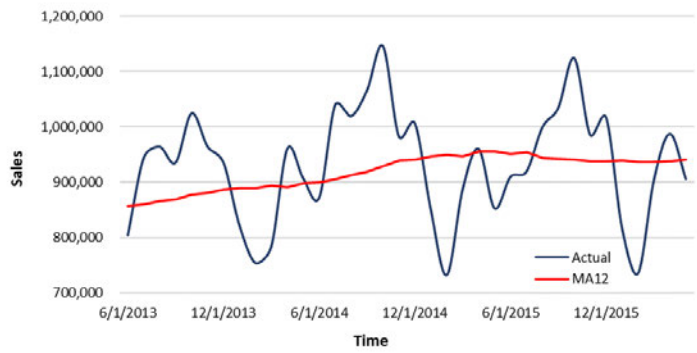


Figure B.1 Dataset 2 and Moving Average Predictions (36 last values)

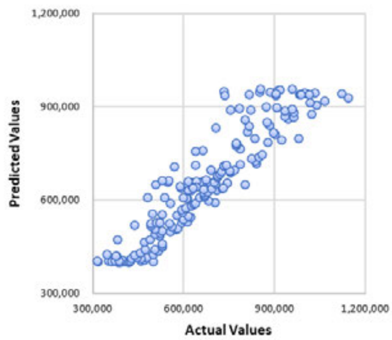


Figure B.2 Correlation between Dataset 2 and Moving Average Predicted Values

Figure B.3 Dataset 2 and Exponential Smoothing Predictions (36 last values)

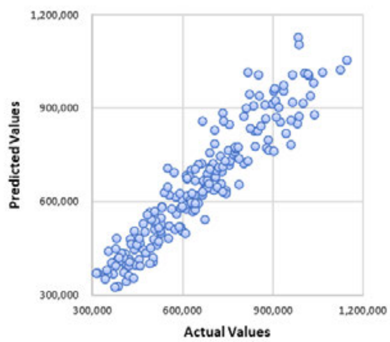


Figure B.4 Correlation between Dataset 2 and Exponential Smoothing Predicted Values

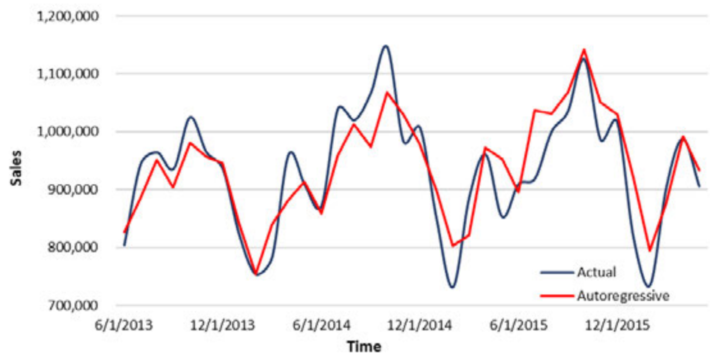


Figure B.5 Dataset 2 and Autoregressive Predictions (36 last values)

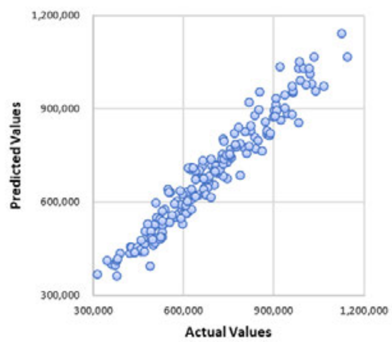


Figure B.6 Correlation between Dataset 2 and Autoregression Predicted Values

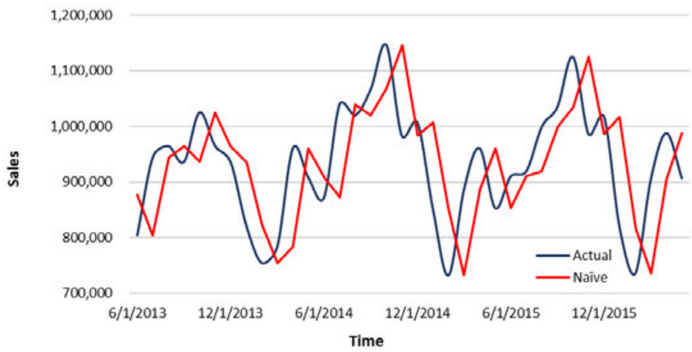


Figure B.7 Dataset 2 and Naive Predictions (36 last values)

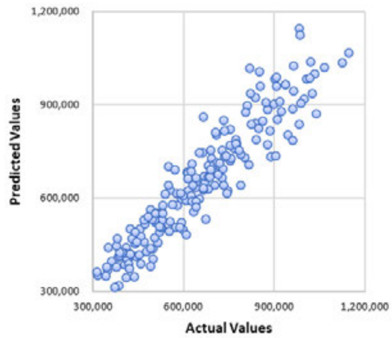


Figure B.8 Correlation between Dataset 2 and Naive Predicted Values

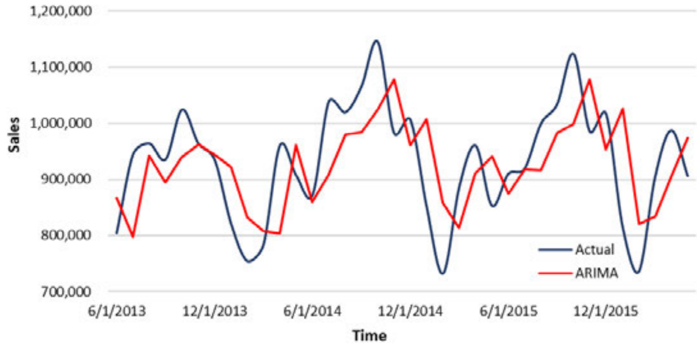


Figure B.9 Dataset 2 and ARIMA Predictions (36 last values)

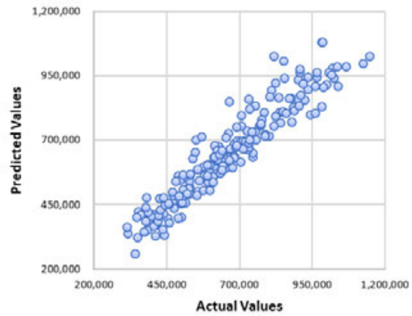


Figure B.10 Correlation between Dataset 2 and ARIMA Predicted Values

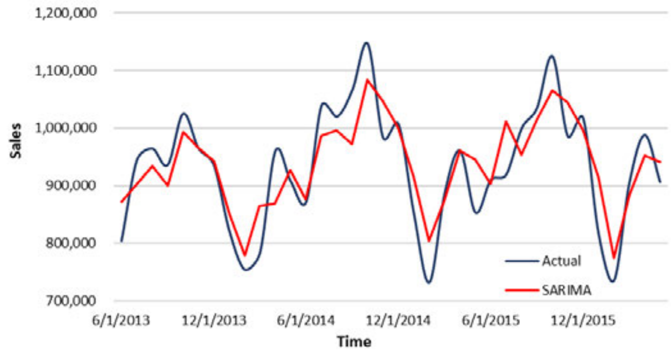


Figure B.11 Dataset 2 and SARIMA Predictions (36 last values)

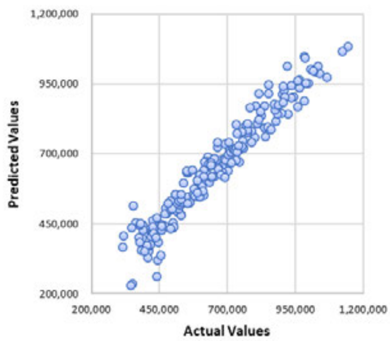


Figure B.12 Correlation between Dataset 2 and SARIMA Predicted Values

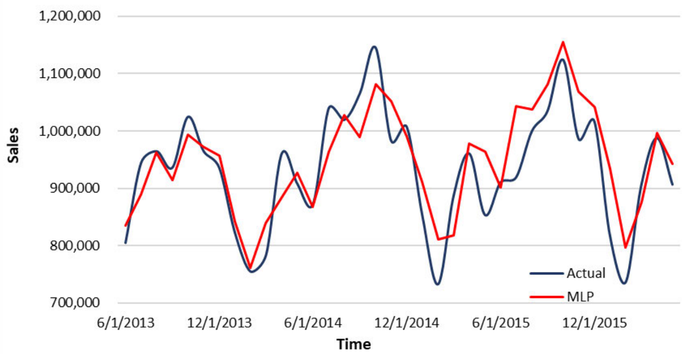


Figure B.13 Dataset 2 and MLP Predictions (36 last values)

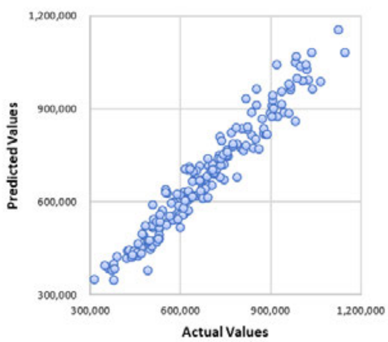


Figure B.14 Correlation between Dataset 2 and MLP Predicted Values

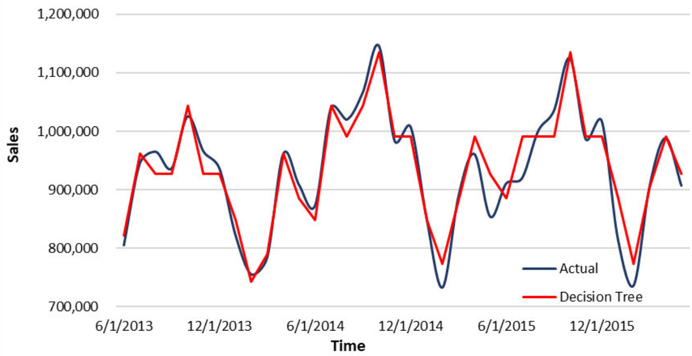


Figure B.15 Dataset 2 and Decision Tree Predictions (36 last values)

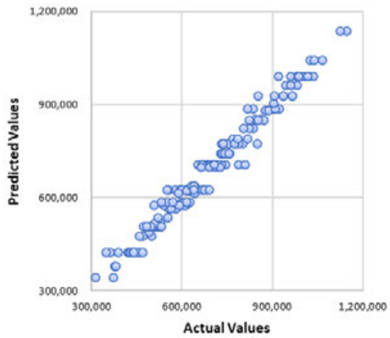


Figure B.16 Correlation between Dataset 2 and Decision Tree Predicted Values

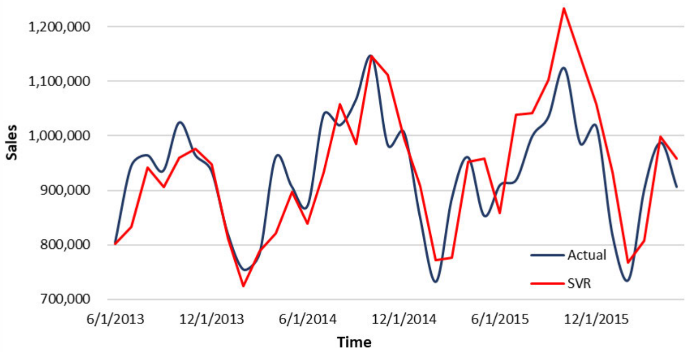


Figure B.17 Dataset 2 and SVR Predictions (36 last values)

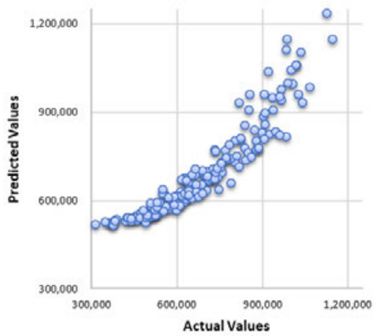


Figure B.18 Correlation between Dataset 2 and SVR Predicted Values

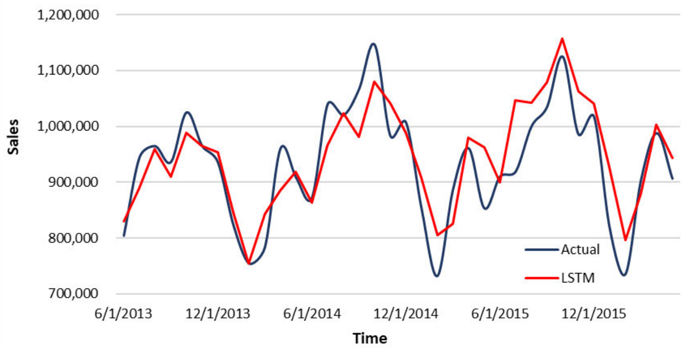


Figure B.19 Dataset 2 and LSTM Predictions (36 last values)

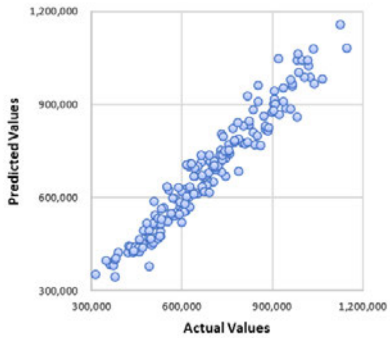


Figure B.20 Correlation between Dataset 2 and LSTM Predicted Values

Plot and Correlation of Dataset 2 - Test Values and Predictions

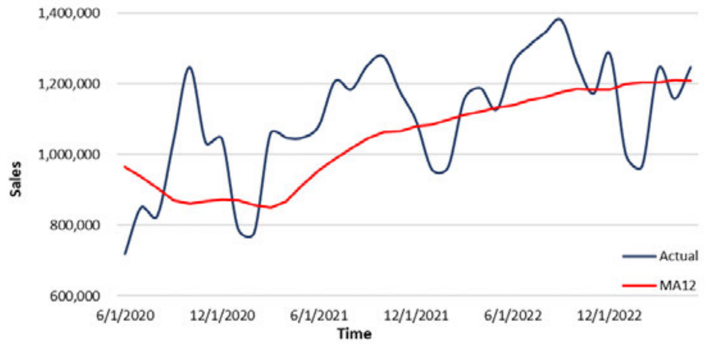


Figure B.21 Dataset 2 and Moving Average Predictions (36 last values)

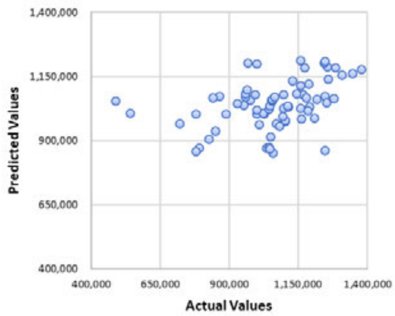


Figure B.22 Correlation between Dataset 2 and Moving Average Predicted Values

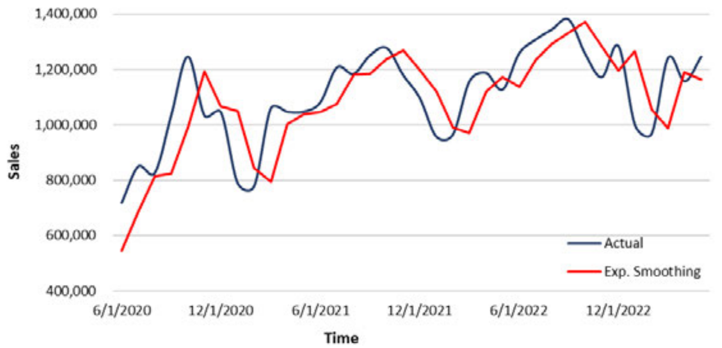


Figure B.23 Dataset 2 and Exponential Smoothing Predictions (36 last values)

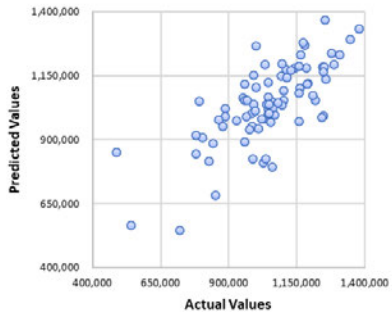


Figure B.24 Correlation between Dataset 2 and Exponential Smoothing Predicted Values

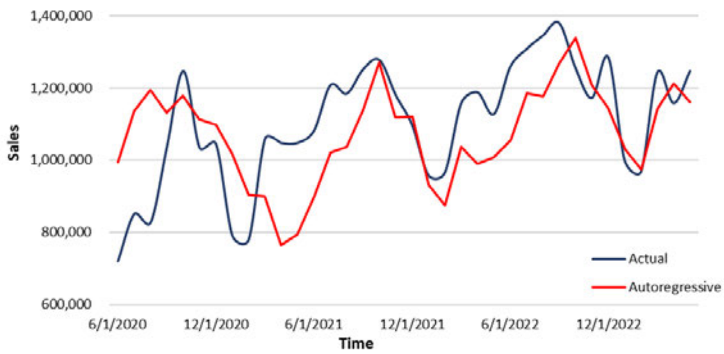


Figure B.25 Dataset 2 and Autoregressive Predictions (36 last values)

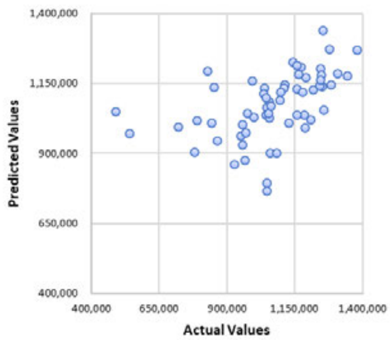
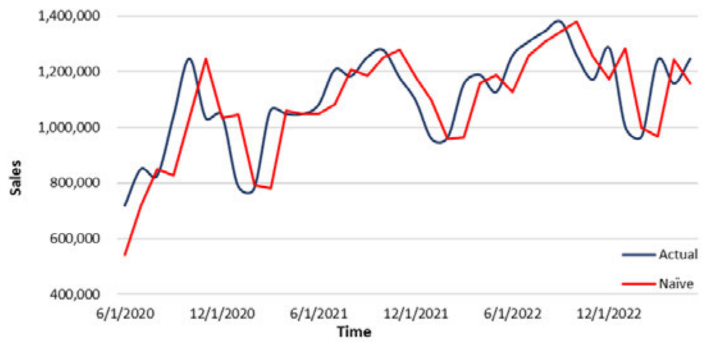


Figure B.26 Correlation between Dataset 2 and Autoregressive Predicted Values



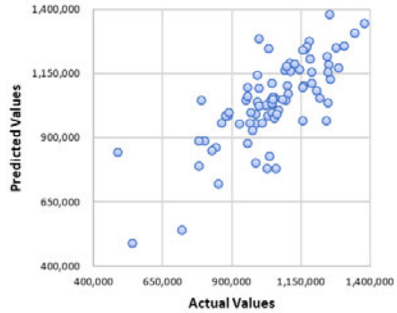
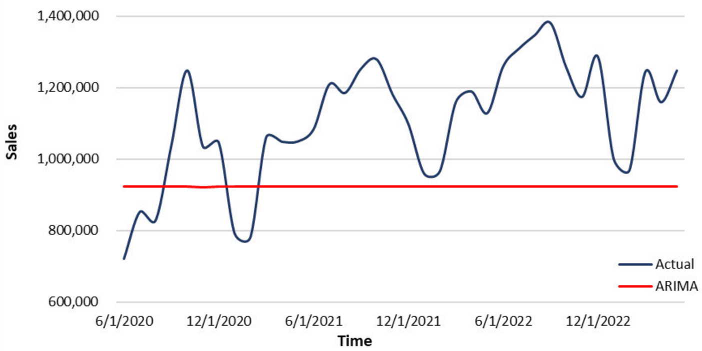


Figure B.27 Dataset 2 and Naive Predictions (36 last values)

Figure B.28 Correlation between Dataset 2 and Naive Predicted Values



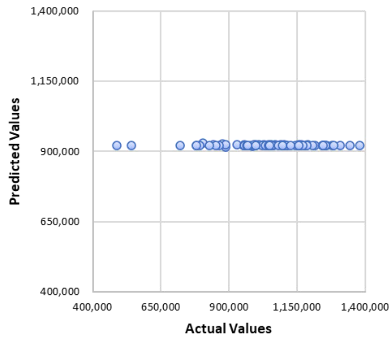
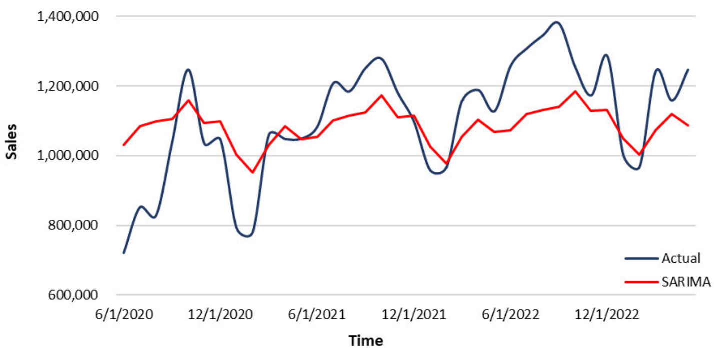


Figure B.29 Dataset 2 and ARIMA Predictions (36 last values)

Figure B.30 Correlation between Dataset 2 and ARIMA Predicted Values



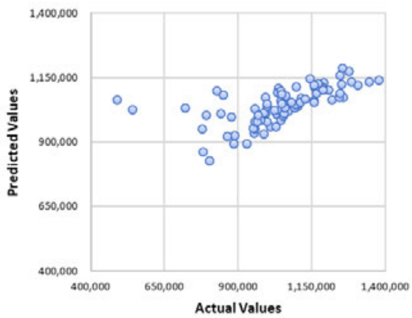
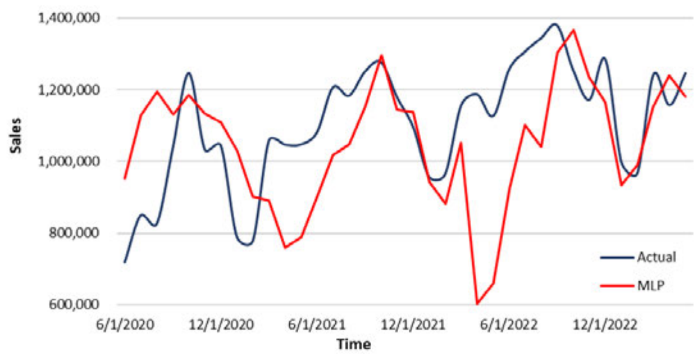


Figure B.31 Dataset 2 and SARIMA Predictions (36 last values)

Figure B.32 Correlation between Dataset 2 and SARIMA Predicted Values



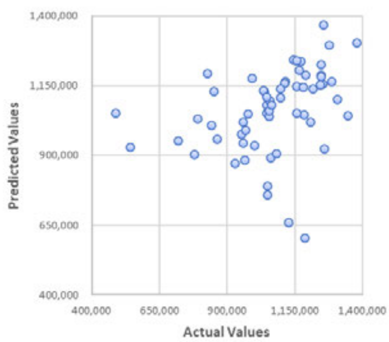


Figure B.33 Dataset 2 and MLP Predictions (36 last values)

Figure B.34 Correlation between Dataset 2 and MLP Predicted Values

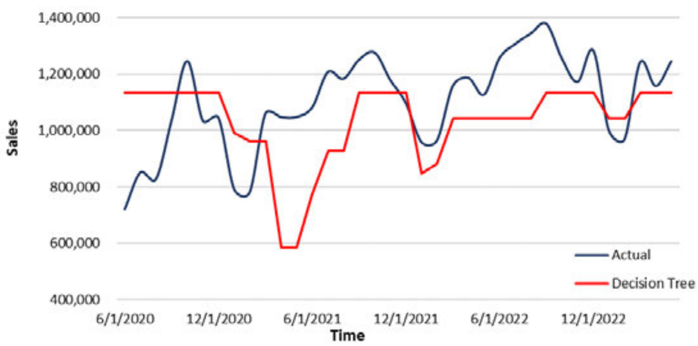


Figure B.35 Dataset 2 and Decision Tree Predictions (36 last values)

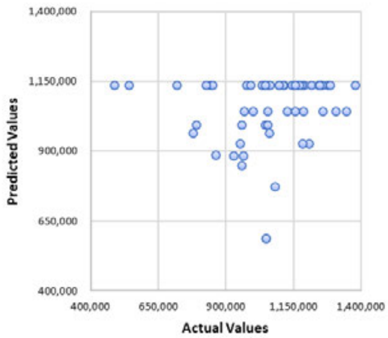


Figure B.36 Correlation between Dataset 2 and Decision Tree Predicted Values

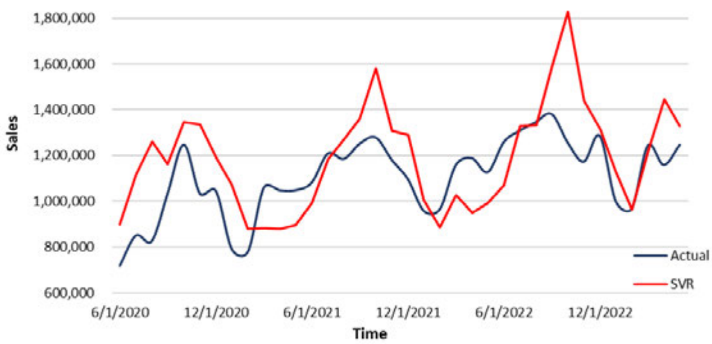


Figure B.37 Dataset 2 and SVR Predictions (36 last values)

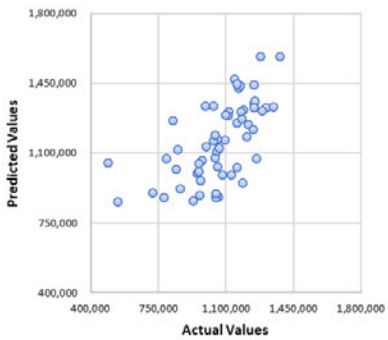


Figure B.38 Correlation between Dataset 2 and SVR Predicted Values

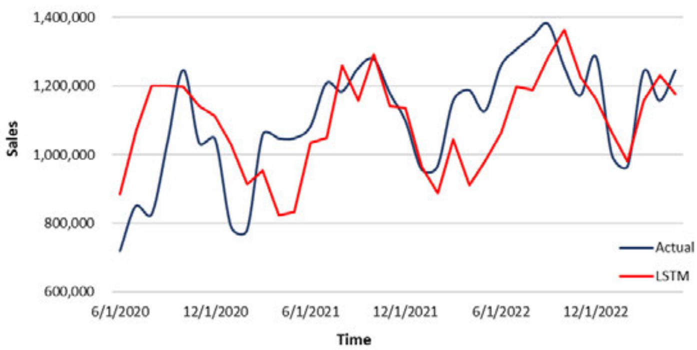


Figure B.39 Dataset 2 and LSTM Predictions (36 last values)

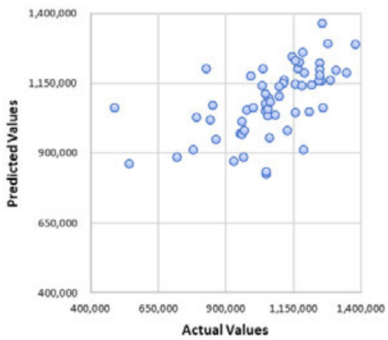


Figure B.40 Correlation between Dataset 2 and LSTM Predicted Values

APPENDIX C

Plot and Correlation of Dataset 3 - Train Values and Predictions

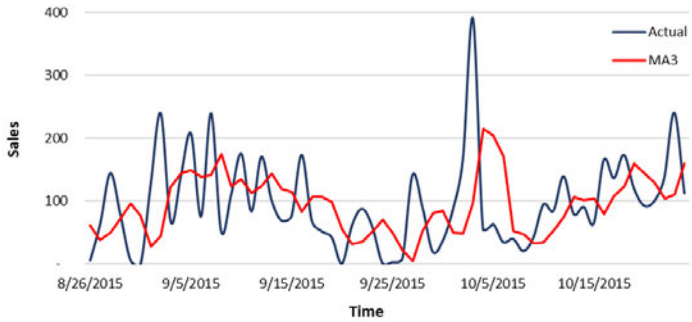


Figure C.1 Dataset 3 and Moving Average Predictions (60 last values)

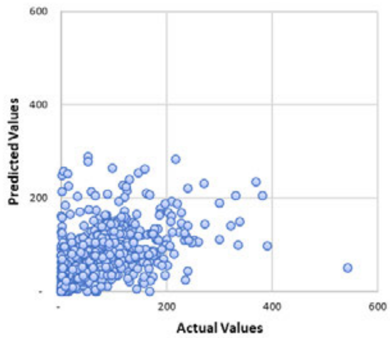


Figure C.3 Correlation between Dataset 3 and Moving Average Predicted Values

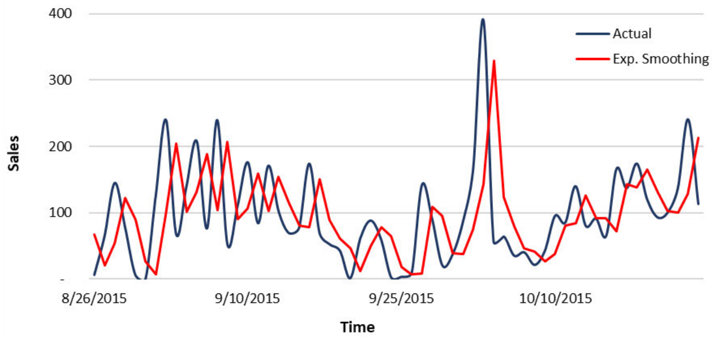


Figure C.3 Dataset 3 and Exponential Smoothing Predictions (60 last values)

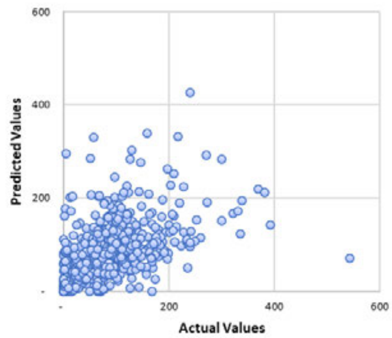


Figure C.4 Correlation between Dataset 3 and Exponential Smoothing Predicted Values

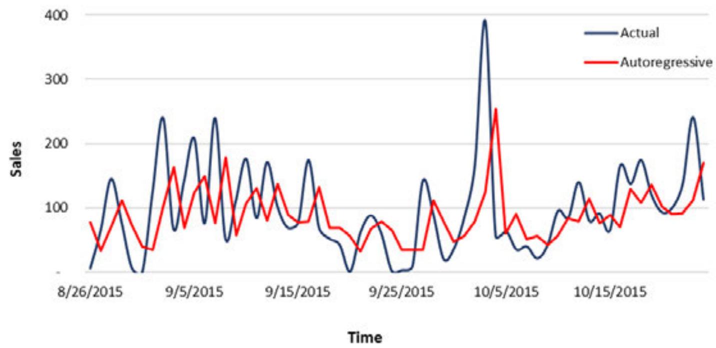


Figure C.5 Dataset 3 and Autoregressive Predictions (60 last values)

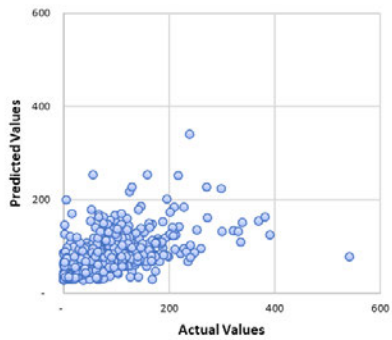


Figure C.6 Correlation between Dataset 3 and Autoregression Predicted Values

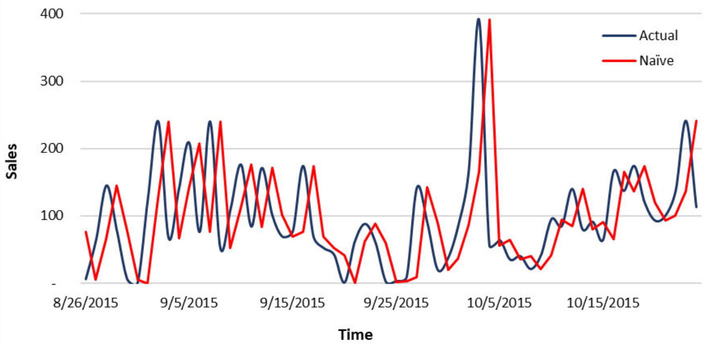


Figure C.7 Dataset 3 and Naive Predictions (60 last values)

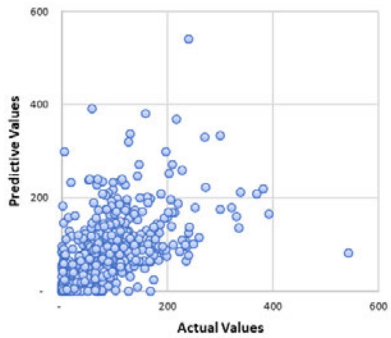


Figure C.8 Correlation between Dataset 3 and Naive Predicted Values

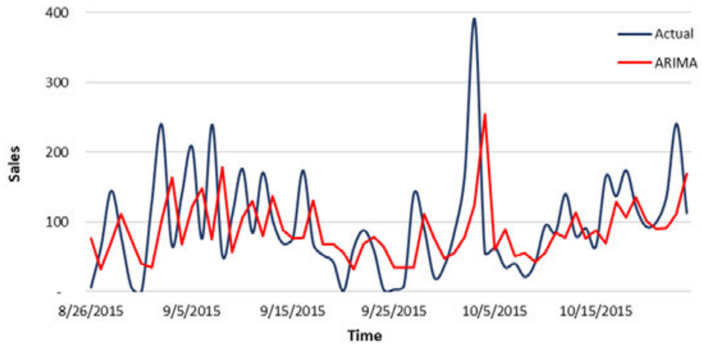


Figure C.9 Dataset 3 and ARIMA Predictions (60 last values)

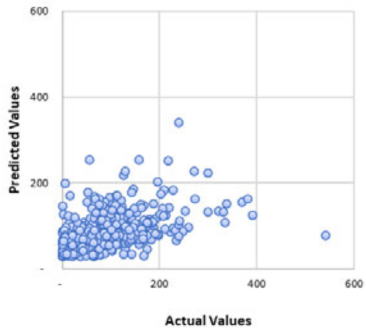


Figure C.10 Correlation between Dataset 3 and ARIMA Predicted Values

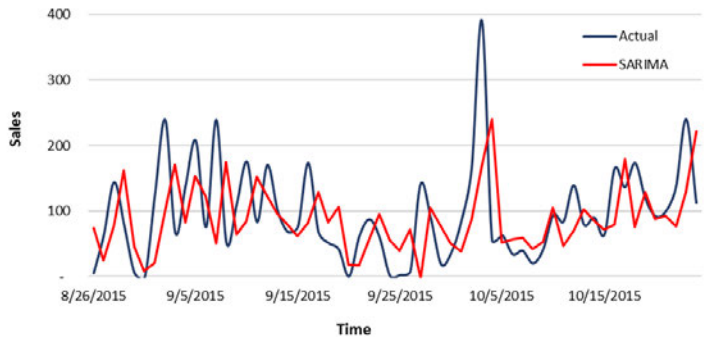


Figure C.11 Dataset 3 and SARIMA Predictions (60 last values)

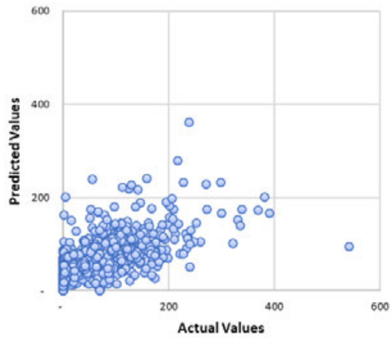


Figure C.12 Correlation between Dataset 3 and SARIMA Predicted Values

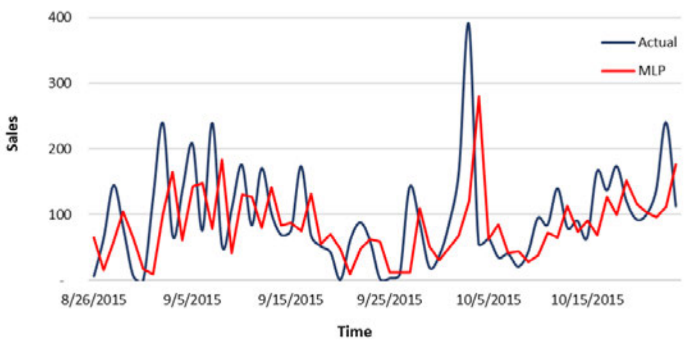


Figure C.13 Dataset 3 and MLP Predictions (60 last values)

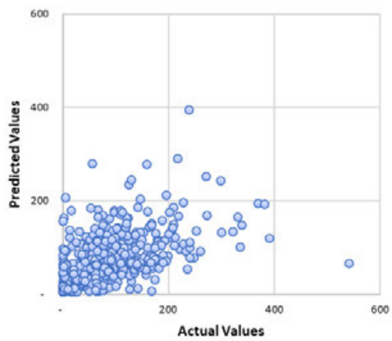


Figure C.14 Correlation between Dataset 3 and MLP Predicted Values

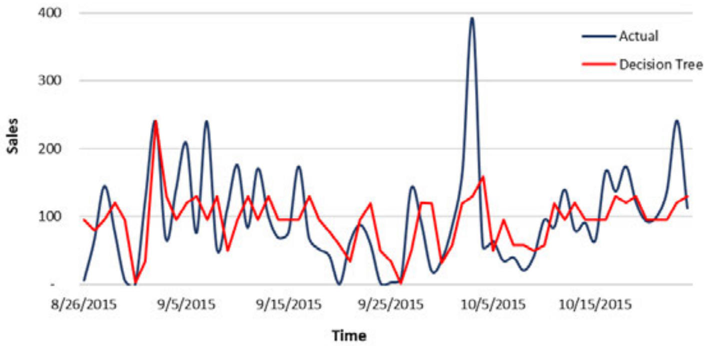


Figure C.15 Dataset 3 and Decision Tree Predictions (60 last values)

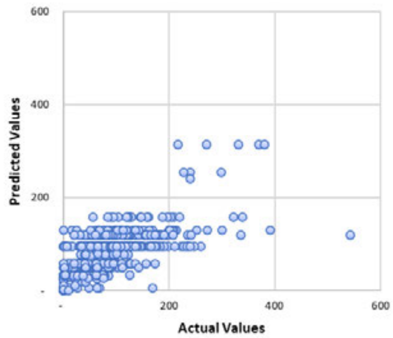


Figure C.16 Correlation between Dataset 3 and Decision Tree Predicted Values

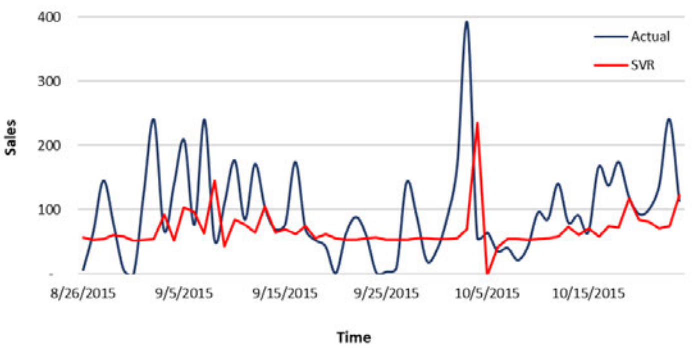


Figure C.17 Dataset 3 and SVR Predictions (60 last values)

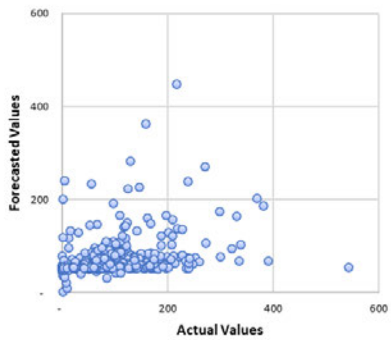


Figure C.18 Correlation between Dataset 3 and SVR Predicted Values

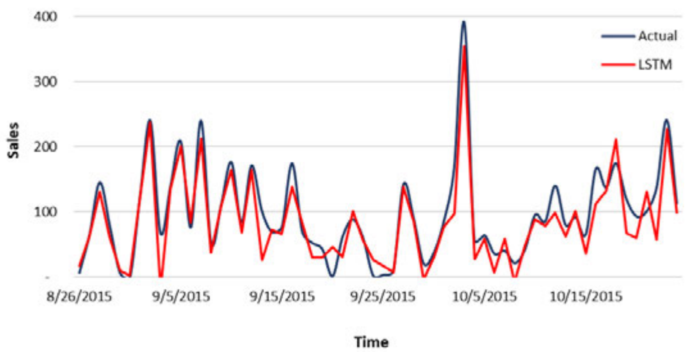


Figure C.19 Dataset 3 and LSTM Predictions (60 last values)

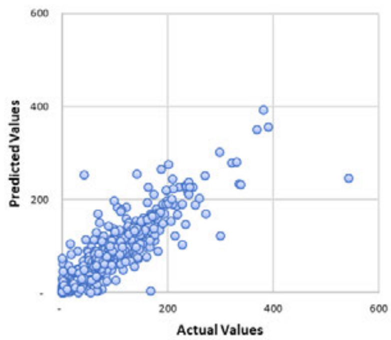


Figure C.20 Correlation between Dataset 3 and LSTM Predicted Values

Plot and Correlation of Dataset 3 - Test Values and Predictions

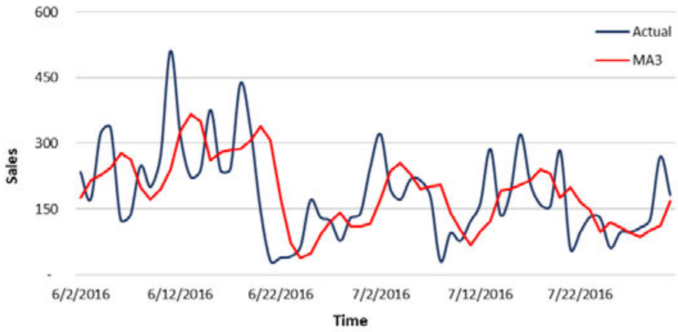


Figure C.21 Dataset 3 and Moving Average Predictions (60 last values)

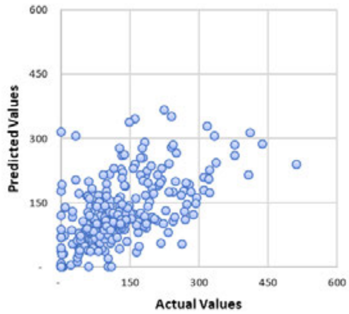


Figure C.22 Correlation between Dataset 3 and Moving Average Predicted Values

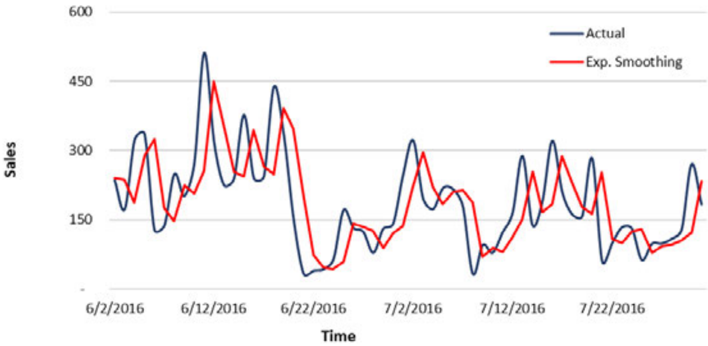


Figure C.23 Dataset 3 and Exponential Smoothing Predictions (60 last values)

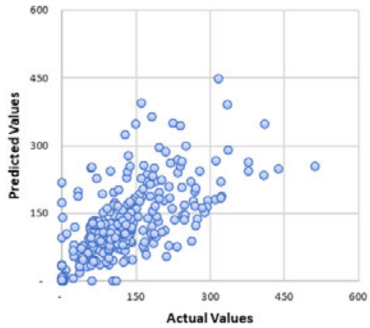


Figure C.24 Correlation between Dataset 3 and Exponential Smoothing Predicted Values

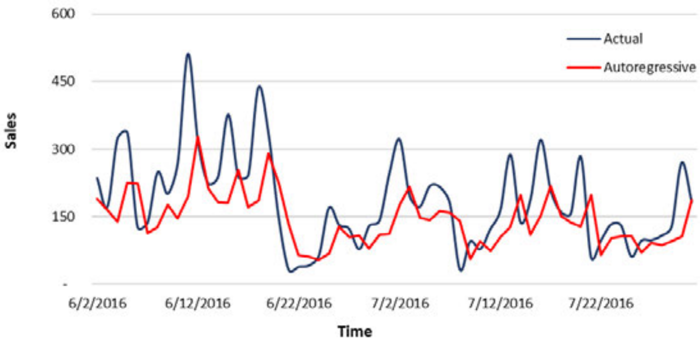


Figure C.25 Dataset 3 and Autoregressive Predictions (60 last values)

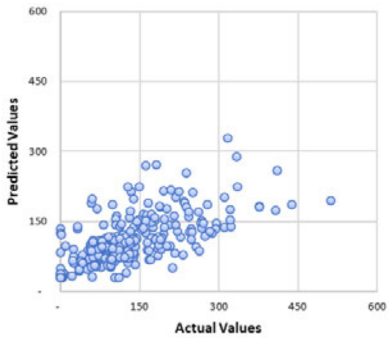


Figure C.26 Correlation between Dataset 3 and Autoregressive Predicted Values

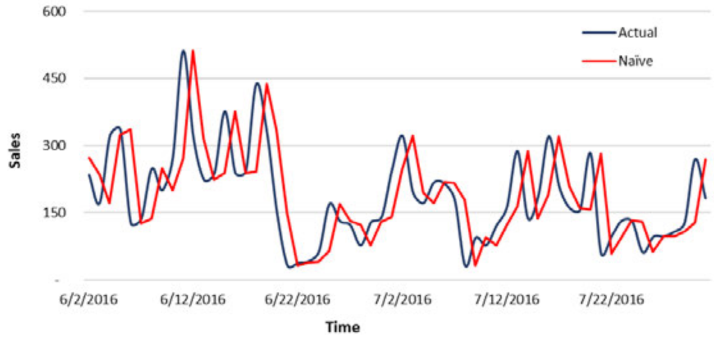


Figure C.27 Dataset 3 and Naive Predictions (60 last values)

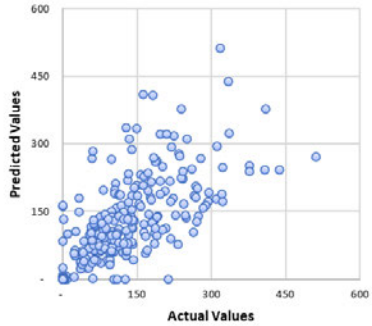


Figure C.28 Correlation between Dataset 3 and Naive Predicted Values

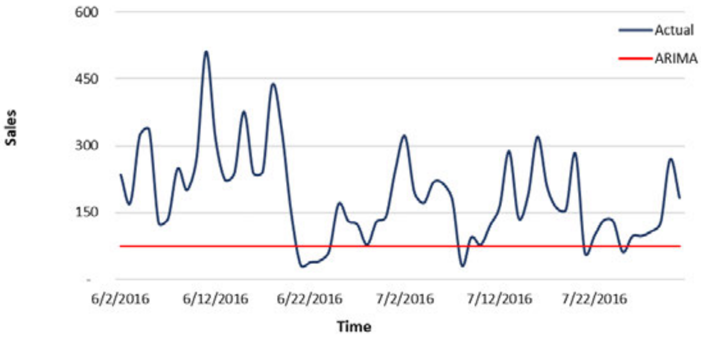


Figure C.29 Dataset 3 and ARIMA Predictions (60 last values)

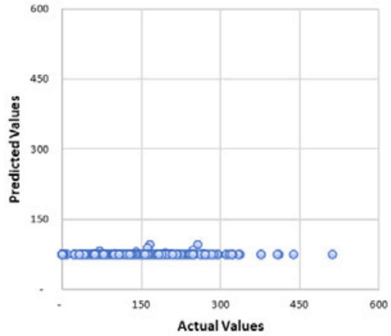


Figure C.30 Correlation between Dataset 3 and ARIMA Predicted Values

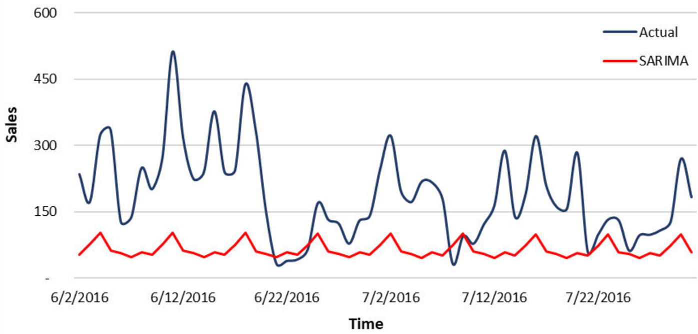


Figure C.31 Dataset 3 and SARIMA Predictions (60 last values)

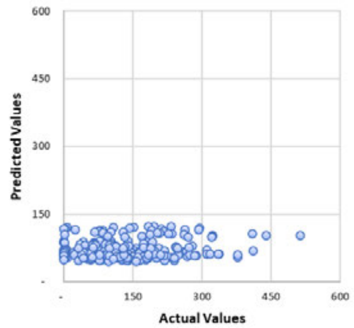


Figure C.32 Correlation between Dataset 3 and SARIMA Predicted Values

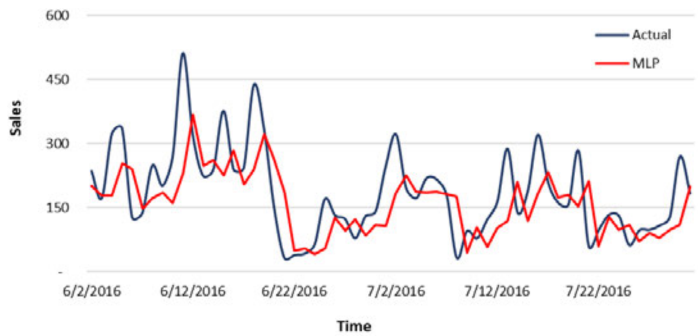


Figure C.33 Dataset 3 and MLP Predictions (60 last values)

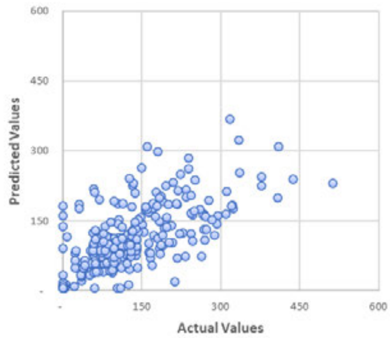
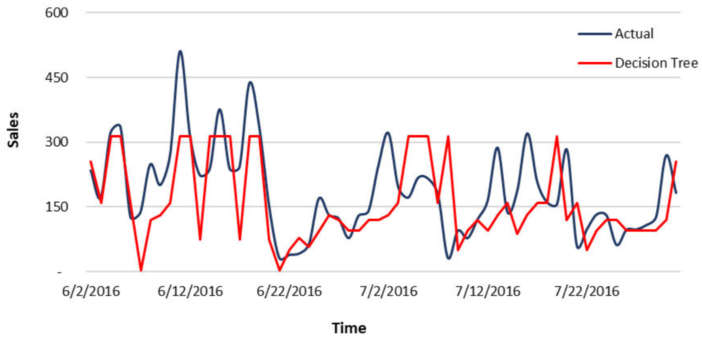


Figure C.34 Correlation between Dataset 3 and MLP Predicted Values



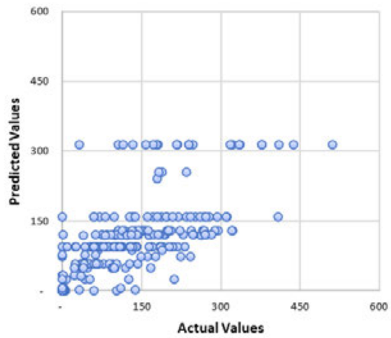
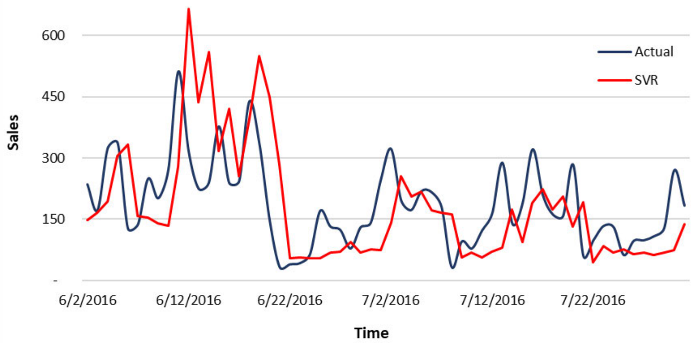


Figure C.35 Dataset 3 and Decision Tree Predictions (60 last values)

Figure C.36 Correlation between Dataset 3 and Decision Tree Predicted Values



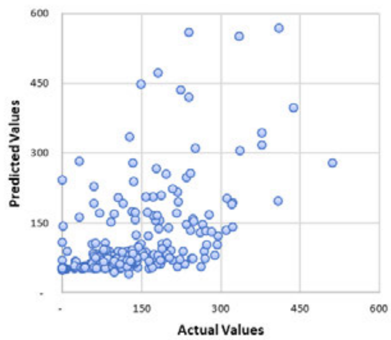
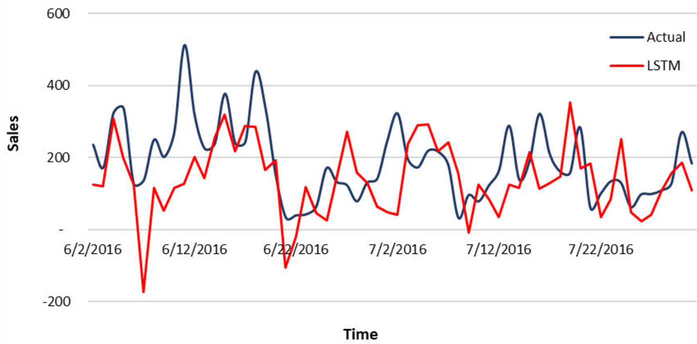


Figure C.37 Dataset32 and SVR Predictions (60 last values)

Figure C.38 Correlation between Dataset 3 and SVR Predicted Values



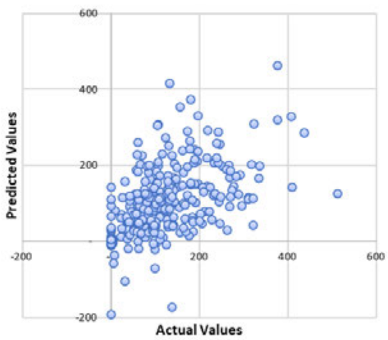


Figure C.39 Dataset 3 and LSTM Predictions (60 last values)

Figure C.40 Correlation between Dataset 3 and LSTM Predicted Values

APPENDIX D

Plot and Correlation of Dataset 4 - Train Values and Predictions

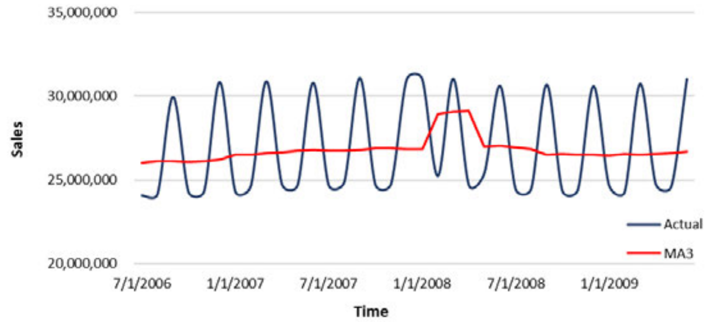


Figure D.1 Dataset 4 and Moving Average Predictions (36 last values)

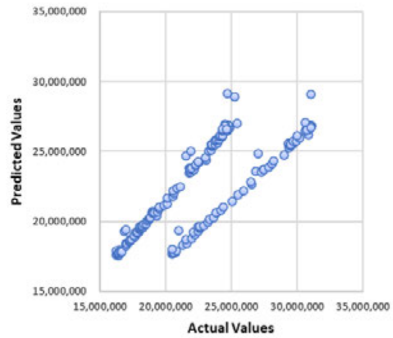


Figure D.2 Correlation between Dataset 4 and Moving Average Predicted Values

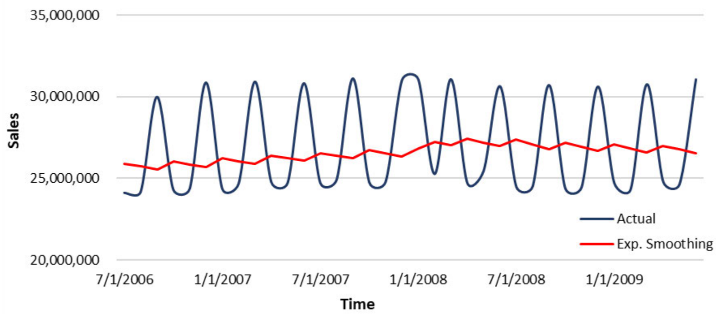


Figure D.3 Dataset 4 and Exponential Smoothing Predictions (36 last values)

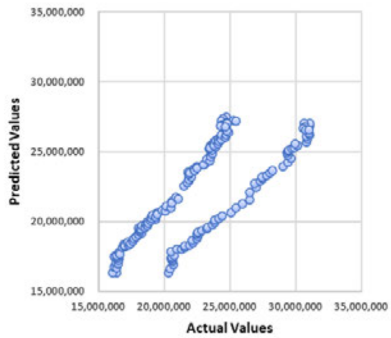


Figure D.4 Correlation between Dataset 4 and Exponential Smoothing Predicted Values

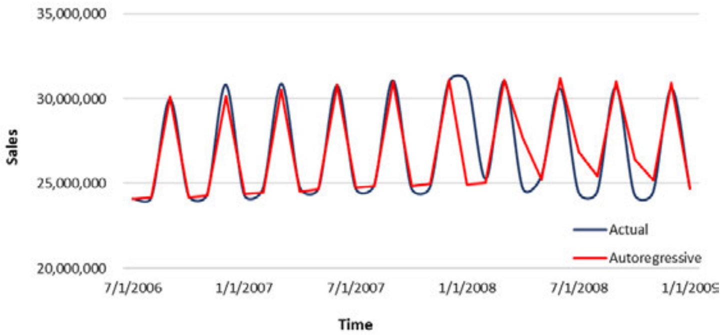


Figure D.5 Dataset 4 and Autoregressive Predictions (36 last values)

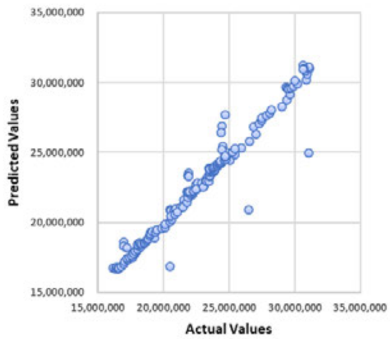


Figure D.6 Correlation between Dataset 4 and Autoregression Predicted Values

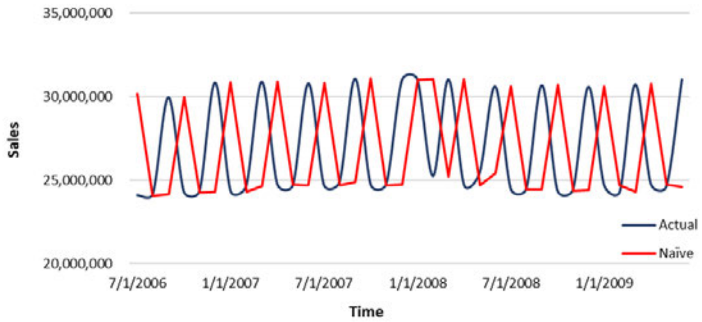


Figure D.7 Dataset 4 and Naive Predictions (36 last values)

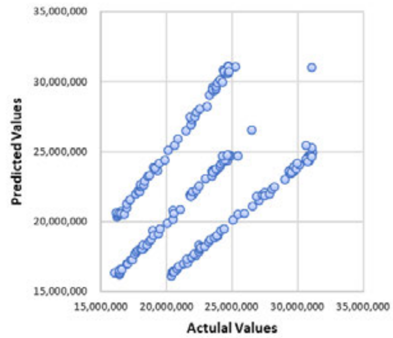


Figure D.8 Correlation between Dataset 4 and Naive Predicted Values

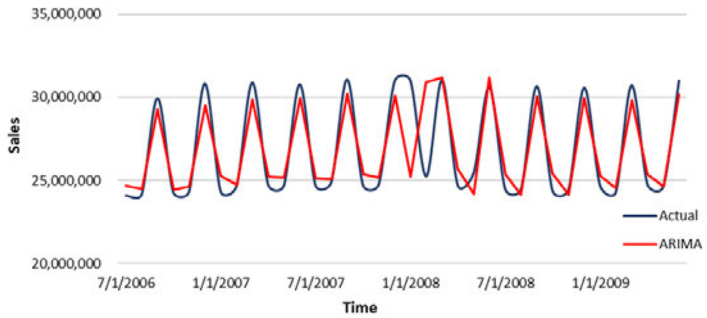


Figure D.9 Dataset 4 and ARIMA Predictions (36 last values)

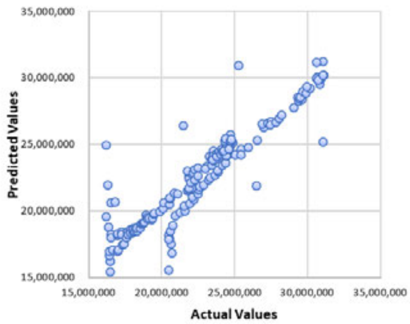


Figure D.10 Correlation between Dataset 4 and ARIMA Predicted Values

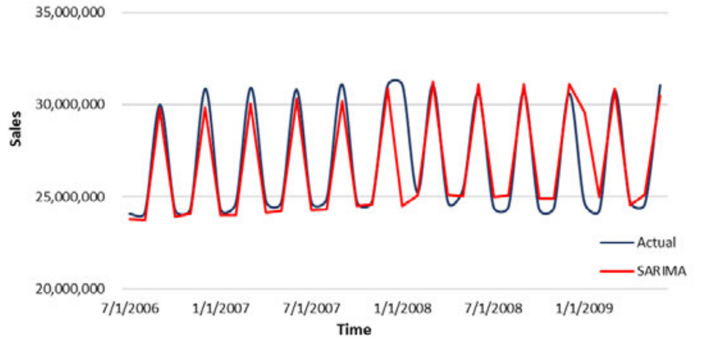


Figure D.11 Dataset 4 and SARIMA Predictions (36 last values)

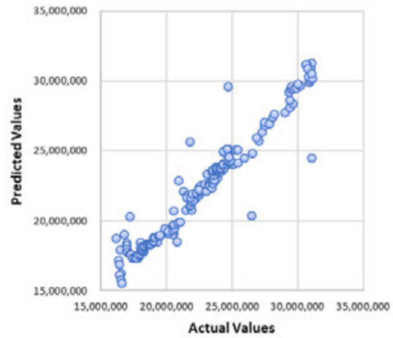


Figure D.12 Correlation between Dataset 4 and SARIMA Predicted Values

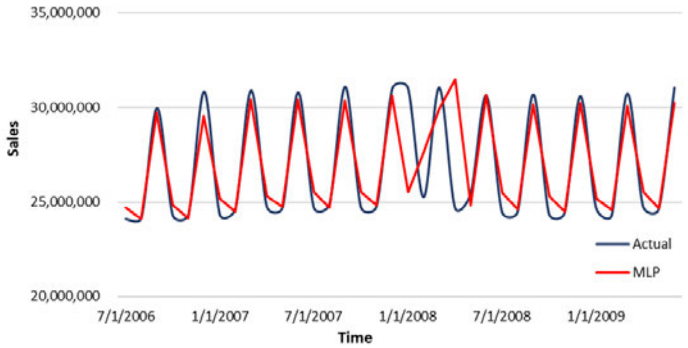


Figure D.13 Dataset 4 and MLP Predictions (36 last values)

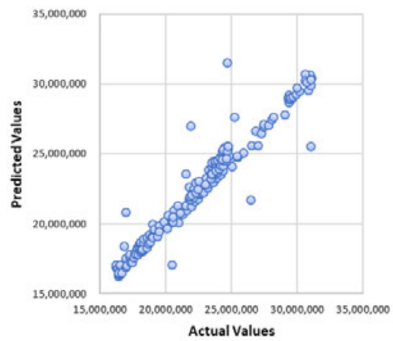


Figure D.14 Correlation between Dataset 4 and MLP Predicted Values

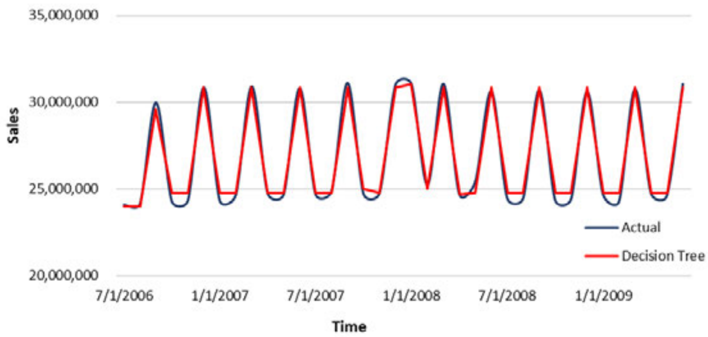


Figure D.15 Dataset 4 and Decision Tree Predictions (36 last values)

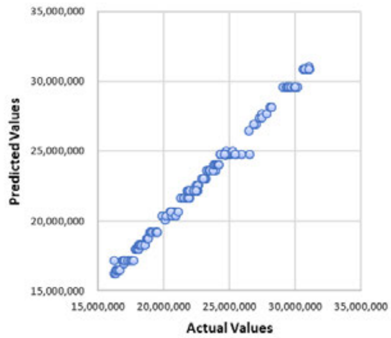


Figure D.16 Correlation between Dataset 4 and Decision Tree Predicted Values

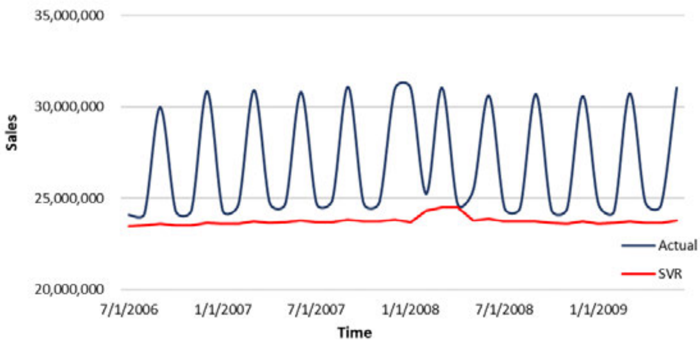


Figure D.17 Dataset 4 and SVR Predictions (36 last values)

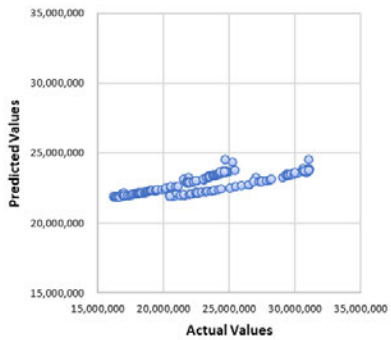


Figure D.18 Correlation between Dataset 4 and SVR Predicted Values

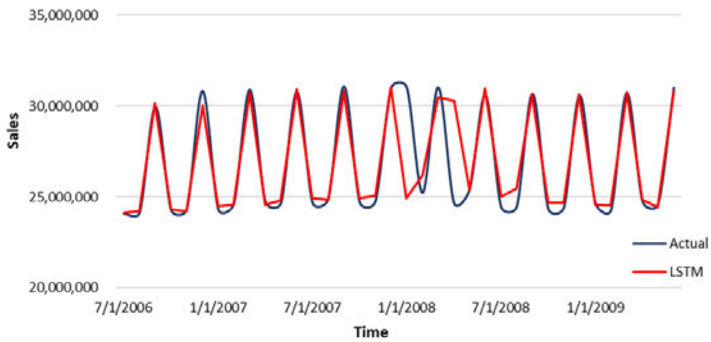


Figure D.19 Dataset 4 and LSTM Predictions (36 last values)

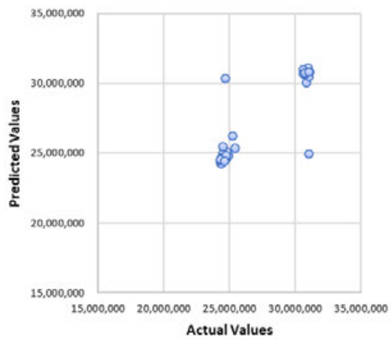


Figure D.20 Correlation between Dataset 4 and LSTM Predicted Values

Plot and Correlation of Dataset 4 - Test Values and Predictions

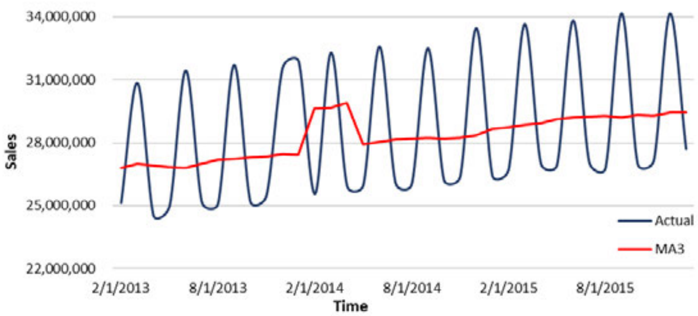


Figure D.21 Dataset 4 and Moving Average Predictions (36 last values)

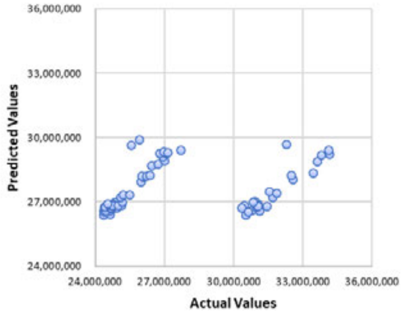


Figure D.22 Correlation between Dataset 4 and Moving Average Predicted Values

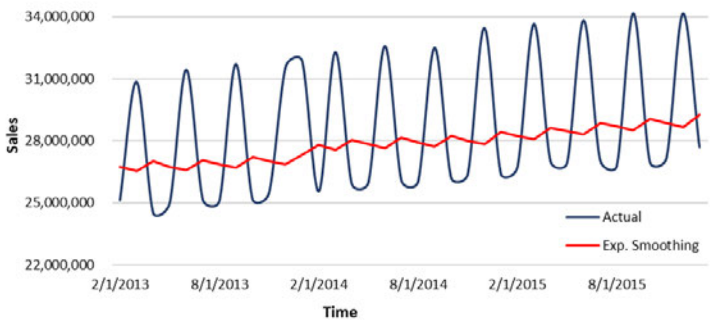


Figure D.23 Dataset 4 and Exponential Smoothing Predictions (36 last values)

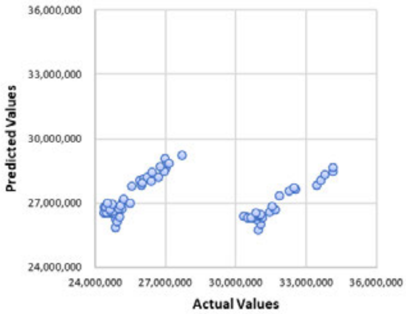


Figure D.24 Correlation between Dataset 4 and Exponential Smoothing Predicted Values

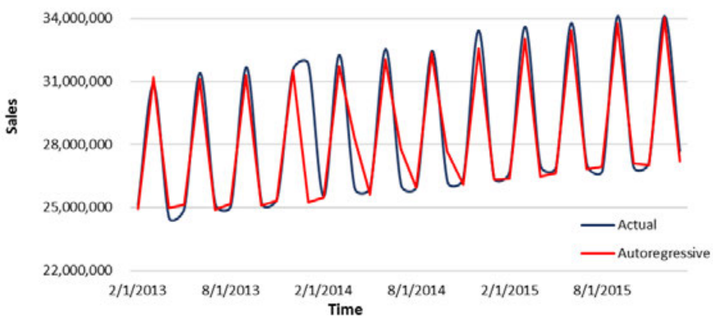


Figure D.25 Dataset 4 and Autoregressive Predictions (36 last values)

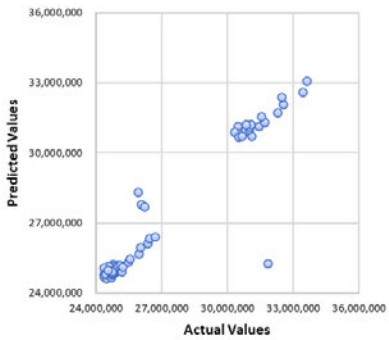


Figure D.26 Correlation between Dataset 4 and Autoregressive Predicted Values

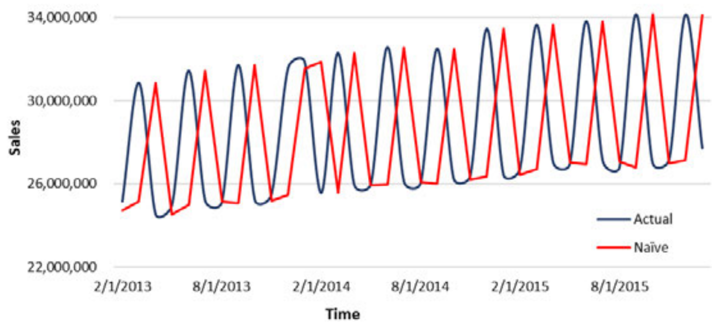


Figure D.27 Dataset 4 and Naive Predictions (36 last values)

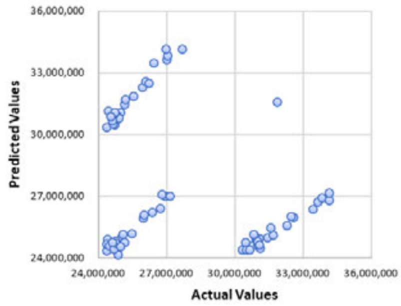


Figure D.28 Correlation between Dataset 4 and Naive Predicted Values

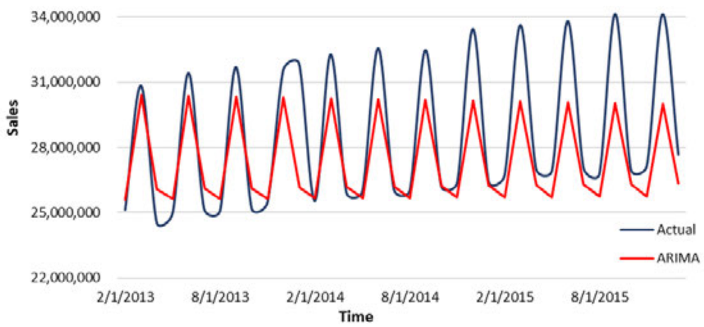


Figure D.29 Dataset 4 and ARIMA Predictions (36 last values)

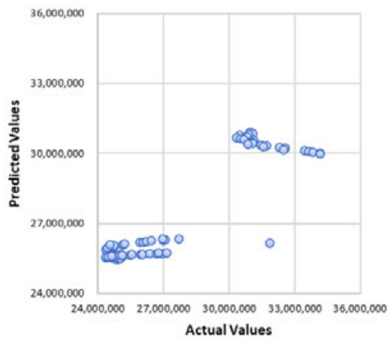


Figure D.30 Correlation between Dataset 4 and ARIMA Predicted Values

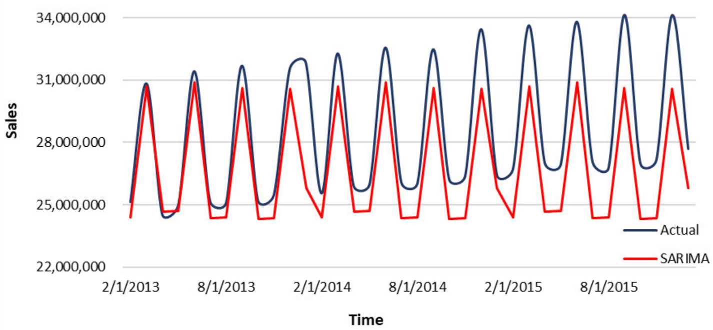


Figure D.31 Dataset 4 and SARIMA Predictions (36 last values)

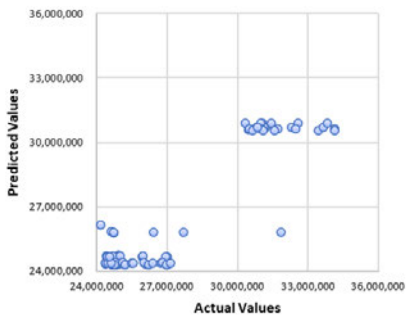


Figure D.32 Correlation between Dataset 4 and SARIMA Predicted Values

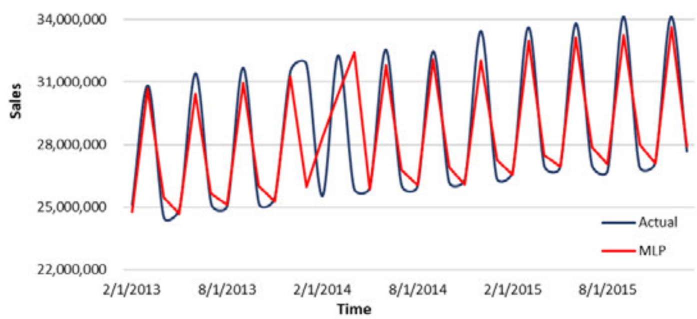


Figure D.33 Dataset 4 and MLP Predictions (36 last values)

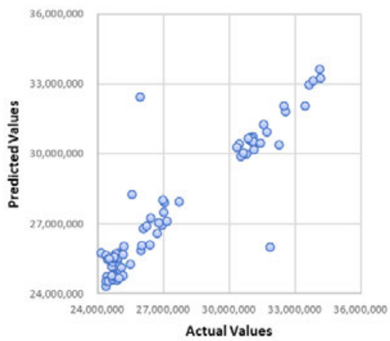


Figure D.34 Correlation between Dataset 4 and MLP Predicted Values

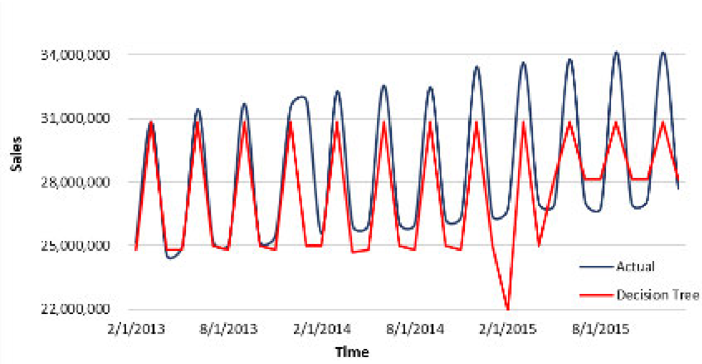


Figure D.35 Dataset 4 and Decision Tree Predictions (36 last values)

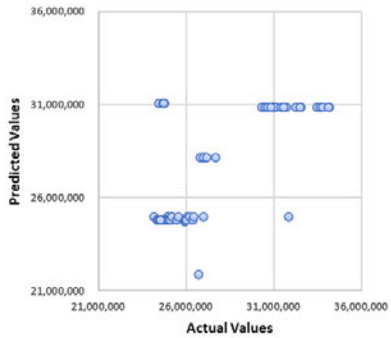


Figure D.36 Correlation between Dataset 4 and Decision Tree Predicted Values

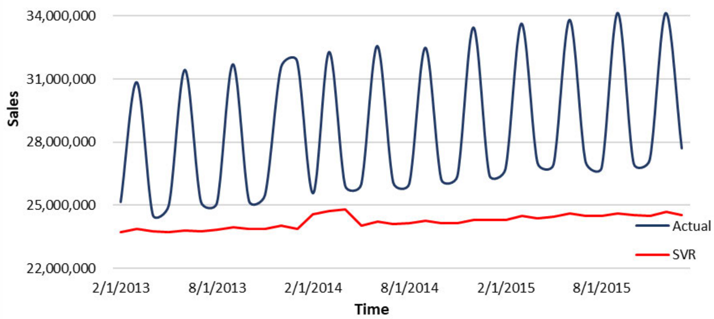


Figure D.37 Dataset 4 and SVR Predictions (36 last values)

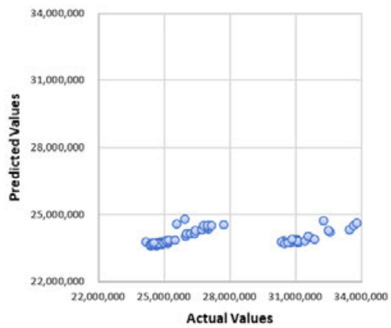


Figure D.38 Correlation between Dataset 4 and SVR Predicted Values

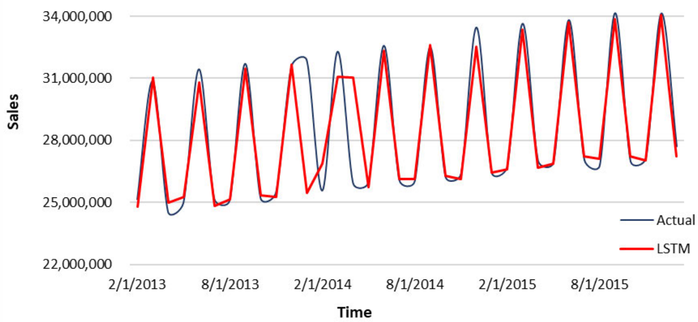


Figure D.39 Dataset 4 and LSTM Predictions (36 last values)

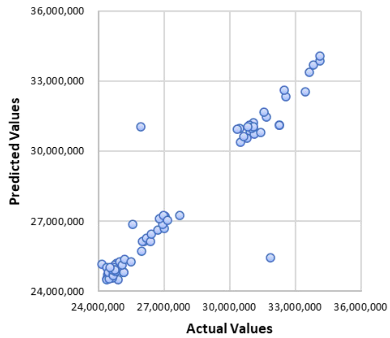


Figure D.40 Correlation between Dataset 4 and LSTM Predicted Values

APPENDIX E

Table E.1 *Correlation and Error Metrics obtained for Dataset 1*

Method	MSE	RMSE	MAE	R^2	Correlation	Peak Similarity
Train Set						
Moving Average	350,902,765	18,732	15,061	0.850	0.922	0.127
Exponential Smoothing	364,457,228	19,091	15,282	0.853	0.923	0.000
Autoregressive	178,699,518	13,368	9,942	0.887	0.942	0.735
Naïve	601,088,072	24,517	19,431	0.762	0.873	0.000
ARIMA	744,139,936	27,279	17,262	0.759	0.871	0.169
SARIMA	635,333,345	25,206	15,235	0.784	0.886	0.331
Multilayer Perceptron	208,396,027	14,436	10,896	0.868	0.941	0.701
Decision Tree	87,577,504	9,358	7,102	0.944	0.972	0.641
Support Vector Regression	199,925,667	14,140	10,585	0.873	0.935	0.675
Long-Short Term Memory	220,763,521	14,858	11,053	0.860	0.936	0.624
Test Set						
Moving Average	855,485,447	29,249	24,594	0.345	0.588	0.111
Exponential Smoothing	877,695,131	29,626	25,123	0.277	0.527	0.000
Autoregressive	802,764,215	28,333	22,635	0.398	0.631	0.667
Naïve	1,206,761,742	34,738	28,204	0.238	0.488	0.000
ARIMA	2,388,334,371	48,871	40,451	0.000	-0.008	0.255
SARIMA	598,985,584	24,474	19,369	0.604	0.777	0.585
Multilayer Perceptron	671,806,769	25,919	19,323	0.385	0.629	0.667
Decision Tree	2,771,724,142	52,647	44,399	-1.537	-0.075	0.179
Support Vector Regression	1,307,716,383	36,162	28,687	-0.197	0.649	0.667
Long-Short Term Memory	655,309,255	25,599	19,666	0.400	0.636	0.641

Table E.2 *Correlation and Error Metrics obtained for Dataset 2*

Method	MSE	RMSE	MAE	R^2	Correlation	Peak Similarity
Train Set						
Moving Average	5,383,329,552	73,371	58,633	0.859	0.927	0.073
Exponential Smoothing	25,431,330,666	67,513	53,194	0.881	0.939	0.000
Autoregressive	4,557,946,136	44,333	34,831	0.940	0.970	0.456
Naïve	13,396,884,770	69,970	54,340	0.880	0.938	0.000
ARIMA	1,965,451,061	67,711	51,063	0.888	0.942	0.275
SARIMA	24,573,233,649	55,450	40,100	0.924	0.961	0.431
Multilayer Perceptron	4,895,776,735	44,607	34,953	0.940	0.970	0.456
Decision Tree	888,048,931	29,800	22,318	0.973	0.986	0.478
Support Vector Regression	4,761,019,932	69,000	54,543	0.855	0.929	0.356
Long-Short Term Memory	2,016,912,094	44,910	35,532	0.939	0.970	0.456
Test Set						
Moving Average	25,431,330,666	159,472	123,593	0.150	0.387	0.027
Exponential Smoothing	4,557,946,136	115,745	90,096	0.536	0.732	0.000
Autoregressive	13,396,884,770	156,759	111,857	0.250	0.500	0.233
Naïve	1,965,451,061	119,107	89,348	0.560	0.748	0.000
ARIMA	24,573,233,649	203,717	171,031	0.002	0.044	0.385
SARIMA	4,895,776,735	129,521	86,268	0.395	0.629	0.500
Multilayer Perceptron	14,186,540,018	187,119	130,763	-0.103	0.354	0.200
Decision Tree	42,218,789,296	205,472	149,315	-0.330	0.127	0.100
Support Vector Regression	39,854,418,586	199,636	156,970	-0.256	0.625	0.300
Long-Short Term Memory	21,379,766,223	146,218	105,259	0.326	0.590	0.267

Table E.3 *Correlation and Error Metrics obtained for Dataset 3*

Method	MSE	RMSE	MAE	R^2	Correlation	Peak Similarity
			Train Set			
Moving Average	4,380	66	44	0.224	0.473	0.158
Exponential Smoothing	3,835	62	40	0.325	0.570	0.000
Autoregressive	3,254	57	41	0.338	0.581	0.032
Naïve	4,133	64	41	0.334	0.578	0.000
ARIMA	3,250	57	41	0.337	0.580	0.031
SARIMA	3,142	56	39	0.364	0.603	0.188
Multilayer Perceptron	3,350	58	38	0.318	0.594	0.057
Decision Tree	2,406	49	33	0.510	0.714	0.098
Support Vector Regression	4,349	66	46	0.114	0.377	0.085
Long-Short Term Memory	1,008	32	19	0.795	0.895	0.688
			Test Set			
Moving Average	6,054	78	57	0.345	0.587	0.023
Exponential Smoothing	5,310	73	52	0.431	0.657	0.000
Autoregressive	5,083	71	51	0.448	0.670	0.044
Naïve	5,622	75	53	0.439	0.663	0.000
ARIMA	11,002	105	76	0.010	0.098	0.000
SARIMA	11,335	106	78	0.034	0.184	0.607
Multilayer Perceptron	4,855	70	49	0.419	0.677	0.068
Decision Tree	5,404	74	52	0.354	0.638	0.090
Support Vector Regression	8,929	94	67	-0.068	0.531	0.083
Long-Short Term Memory	8,716	93	69	-0.042	0.504	0.316

Table E.4 *Correlation and Error Metrics obtained for Dataset 4*

Description	MSE	RMSE	MAE	R^2	Correlation	Peak Similarity
			Train			
Moving Average	6,405,591,436,236	2,530,927	2,324,985	0.629	0.793	0.150
Exponential Smoothing	7,278,920,766,915	2,697,948	2,299,969	0.599	0.774	0.000
Autoregressive	711,733,243,340	843,643	377,052	0.958	0.979	0.761
Naive	18,017,971,388,424	4,244,758	3,465,873	0.227	0.477	0.000
ARIMA	4,206,419,621,162	2,050,956	1,101,831	0.782	0.884	0.610
SARIMA	15,135,797,491,007	3,890,475	1,598,260	0.649	0.805	0.642
Multilayer Perceptron	1,152,329,432,707	1,073,466	597,236	0.933	0.966	0.633
Decision Tree	77,769,857,766	278,872	189,849	0.995	0.998	0.600
Support Vector Regression	13,063,263,064,206	3,614,314	2,839,404	0.242	0.810	0.733
Long-Short Term Memory	822,337,568,033	906,828	342,954	0.952	0.976	0.758
			Test			
Moving Average	9,382,766,623,955	3,063,130	2,860,426	0.082	0.287	0.204
Exponential Smoothing	10,632,533,278,564	3,260,757	2,866,944	0.016	0.125	0.000
Autoregressive	875,828,216,859	935,857	422,850	0.915	0.957	0.711
Naive	27,154,703,282,156	5,211,017	4,309,805	0.106	-0.325	0.000
ARIMA	2,164,370,179,638	1,471,180	1,048,673	0.839	0.916	0.706
SARIMA	2,268,999,820,563	1,506,320	1,040,379	0.851	0.922	0.765
Multilayer Perceptron	1,514,201,441,978	1,230,529	693,868	0.852	0.925	0.673
Decision Tree	4,135,212,548,222	2,033,522	1,138,086	0.595	0.784	0.490
Support Vector Regression	22,091,323,687,702	4,700,141	3,561,140	-1.164	0.428	0.653
Long-Short Term Memory	1,010,602,178,821	1,005,287	399,077	0.901	0.949	0.735